Advances in Engineering Software 42 (2011) 660–670

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

1D unified mathematical model for environmental flow applied to steady aerated mixed flows

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ARTICLE INFO

Article history:
Received 18 September 2009
Received in revised form 27 April 2011
Accepted 28 April 2011
Available online 28 May 2011

Keywords:
Finite volume method
Homogeneous Equilibrium Model
Preissmann slot
Local Instant Formulation
Air-entrainment
Civil engineering
Hydraulic engineering

ABSTRACT

Hydraulic models available in literature are unsuccessful in simulating accurately and efficiently *environmental flows* characterized by the presence of both air–water interactions and free-surface/pressurized transitions (aka mixed flows). The *purpose* of this paper is thus to fill this knowledge gap by developing a unified one-dimensional mathematical model describing free-surface, pressurized and mixed flows with air–water interactions. This work is part of a general research project which aims at establishing a unified mathematical model suitable to describe the vast majority of flows likely to appear in civil and environmental engineering (pure water flows, sediment transport, pollutant transport, aerated flows…). In order to tackle this problem, our original *methodology* consists in both time- and space-averaging the Local Instant Formulation, which includes field equations for each phase taken separately and jump conditions, over a flow cross-section involving a free-surface. Subsequently, applicability of the model is extended to pressurized flows as well. The first key *result* is an original 1D Homogeneous Equilibrium Model which describes two-phase free-surface flows. It is proven to be fundamentally multiphase, to take into account scale heterogeneities of environmental flow and to be very easy to solve. Next, applicability of this free-surface model is extended to pressurized flows by using the classical Preissmann slot concept. A second key result here is the introduction of an original negative Preissmann slot to simulate sub-atmospheric pressurized flows. The model is then closed by using constitutive equations suitable for air–water flows. Finally, this mathematical model is discretised by means of a finite volume scheme and validated by comparison with experimental results from a physical model in the case of a steady flow in a large scale gallery.

1. Introduction

Civil and environmental engineers make frequent use of mathematical and numerical models to handle with hydraulics problems. In this respect, the need for consistent mathematical and numerical models has never been more pressing. The acuteness of the situation is prompted by growing concerns about ecological, technical and economic issues. As an evocative example of such a hydraulic problem, one can cite mixed flows, characterized by the simultaneous occurrence of free-surface and pressurized flows (Fig. 1). This flow pattern is frequently encountered in rivers networks (water intakes and deviations in closed pipes), sewer systems, storm-water storage pipes, flushing galleries, bottom outlets, … As a matter of fact, some hydraulic structures are designed to combine free-surface and pressurized sections (e.g. water intakes). In addition, dynamic pipe filling bores may occur in hydraulic structures designed only for conveying free-surface flow under an extreme water inflow or upon starting a pump [1,2]. During such a transition, highly transient phenomena appear and may cause structural damages to the system [3], generate geysers through vertical shafts [4], engender flooding, … What is more, air/water interactions may arise, particularly at the transition bore [5], and alter thoroughly the flow regime and its characteristics. On account of the range of applications affected by mixed flows, a good prediction of its features is an industrial necessity.

Scientific literature offers different mathematical approaches to describe mixed flows. First, the so-called *shock-tracking approach* consists in solving separately free-surface and pressurized flows through different sets of equations [6,7]. The advantage of this method is that the transition is computed as a true discontinuity (infinite resolution). Such an algorithm is very complicated and case-specific so that it is difficult to apply it to practical applications. Important experimental information on the transition motion is given by Cardle and Song [6]. As a particular case of shock-tracking approach, the Rigid Water Column Approach [7]

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doi:10.1016/j.advengsoft.2011.04.012

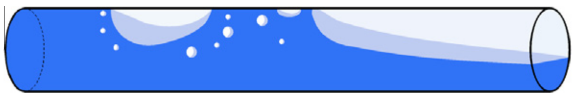

Fig. 1. Mixed flow typical configuration involves an air–water pressurized flow and a free-surface flow separated by a moving transition.

treats each phase (air/water) separately on the basis of a specific set of equations. The latter approach succeeds in simulating complex configurations of the transition but fails in its attempt to describe all flow regimes. Using the method for practical application is also difficult because of the complexity and specificity of the algorithm. Second, the so-called *shock-capturing* approach is a family of method which computes pressurized and free-surface flows by using a single set of equations [8–11]. The most widespread of these methods is the Preissmann slot [8,12] because it only uses the classical Saint–Venant equations [13]. Such a model is however unable to simulate sub-atmospheric pressurized flows [14]. Recently, research focuses to integrate the effect of the air phase on the behavior of mixed flows. To authors' knowledge, two methods integrate the effect of pressurization of the air phase above the free-surface: the Rigid Water Column [7] and the shock-capturing model of Vasconcelos [9,15]. However, these models do not account for the dispersed air in the water flow (air bubbles and pockets).

On account of this literature review, one can say that present models fail to simulate the presence of dispersed air in the water and free-surface/pressurized flow in a unified framework. The purpose of this paper is thus to fill this knowledge gap by developing a unified one dimensional mathematical model describing free-surface, pressurized and mixed flows with air entrainment. This work is part of a general research project which aims at establishing a unified mathematical model suitable to describe the vast majority of flows likely to appear in civil and environmental engineering. It includes pure-water flows, sediment transport over a mobile bed, pollutant transport, aerated flows… Four conditions are sought in the development of the model:

- The model must take into account accurately the motion of a dispersed air phase in water flow, and in particular it should describe efficiently the interaction of the water flow with the dispersed phase and the external environment.
- The model must handle correctly the scale heterogeneities in time and space, which are characteristic of practical applications and mechanisms encountered in free-surface and pressurized hydraulics.
- The model must treat in a unified framework mixed flows, characterized by the simultaneous occurrence of free-surface and pressurized flows.
- The model must require a moderate computational effort to solve most of practical cases in civil and environmental engineering (such that 3D models are not considered here).

Therefore, this paper proposes an original 1D Homogeneous Equilibrium Model (HE-Model) for free-surface flows whose applicability is extended to pressurized and mixed flows by means of the classical Preissmann slot and an original negative Preissmann slot. This model is proven to meet the previous objectives in many respects. The paper is divided in two parts. The first one exposes the derivation of the original mathematical model and the numerical scheme used to solve it. In the second part, we present the application of this new model to the case of flows in a gallery. Experimental results from a physical model build in the Laboratory of Engineering Hydraulics of the University of Liege are used for comparison with numerical results.

2. Unified mathematical model

2.1. Three-dimensional homogeneous flow model

If we assume that each sub-region bounded by interfaces in an air–water flow may be considered as a continuum, the standard single-phase Navier–Stokes equations holds for each sub-region with appropriate jump and boundary conditions. This is the Local Instant Formulation (LIF) which is extensively derived and commented in [16,17]. In principle, a two-phase flow model should solve the Local Instant Formulation. Obtaining a solution this way is however mathematically difficult and beyond the present computational capability for many engineering applications. On account of this, practical model have been developed. Most of them are derived by application of an averaging procedure on the LIF. In the present work, the Eulerian time averaging procedure is chosen because it is proven to be particularly useful for turbulent two-phase flow. Mathematical operation lead to the drift-flux model [18]. In this method, it is assumed that the multiphase flow may be described as a single phase flow of mixture variables that refer to the motion of the center of mass. The motion of the dispersed phase is then treated in terms of diffusion through the mixture. Since the momentum equation for this phase is neglected, a constitutive equation for the relative velocity is required.

In particular, the drift-flux model simplifies into the Homogeneous Equilibrium Model (HE-Model) if all phases are assumed to move at the same velocity (the relative velocity is negligible). The model is commonly used for the simulation of heat exchangers [19,20], two-phase flow in ducts [21],… For further details, we refer the interested reader to the classical book of Ishii and Hibiki [16]. The resulting field equations contain a continuity equation, a diffusion equation and a momentum equation:

$$\begin{cases} \frac{\partial \rho_m}{\partial t} + \nabla(\rho_m \mathbf{v}_m) = 0 \\ \frac{\partial \alpha_g}{\partial t} + \nabla(\alpha_g \mathbf{v}_m) = \Gamma_g \\ \frac{\partial \rho_m \mathbf{v}_m}{\partial t} + \nabla(\rho_m \mathbf{v}_m \mathbf{v}_m) = -\nabla p_m + \nabla(\tau_m + \tau^T) + \rho_m \mathbf{g} + \mathbf{M}_m \end{cases} \tag{1}$$

where ρ_m [kg/m^3] is the mixture density, $\mathbf{v}_m$ [ms^{-1}] is the mixture velocity vector (under the assumption of velocity equilibrium, $\mathbf{v}_m = \mathbf{v}_{\text{water}} = \mathbf{v}_{\text{air}}$), α_g [–] is the air void fraction, Γ_g [s^{-1}] is the phase change volume generation, p_m [Nm^{-2}] is the mixture pressure, τ_m [Nm^{-2}] and τ^T [Nm^{-2}] are the viscous and turbulent stress tensors, g [ms^{-2}] is the gravity and $\mathbf{M}_m$ [kg s^{-2} m^{-2}] is the interfacial momentum source. It is worthwhile noting that the simplicity of Eq. (1) results from the wise choice of the mixture macroscopic properties (i.e. mixture center of mass velocity, mixture density, mixture pressure…).

Closure of the HE-model requires the definition of the mixture variables and a constitutive equation. Air and water are supposed to be incompressible Newtonian fluids. The assumption of incompressibility may seem inappropriate, especially for pressurized flows. Hopefully, the compressibility of both fluids is accounted for a posteriori when extending applicability of the free-surface model to pressurized flows (see Section 2.4). The value of the celerity takes indeed into account the compressibility of the fluid. Consequently, the mixture properties are written as:

$$\begin{aligned} \rho_m &= \alpha_g \rho_g + (1-\alpha_g)\rho_w \cong (1-\alpha_g)\rho_w \\ \tau_m &= [\alpha_g \mu_g + (1-\alpha_g)\mu_w](\nabla \cdot \mathbf{v} + (\nabla \cdot \mathbf{v})^T) \end{aligned} \tag{2}$$

At this point, no assumption is needed for the constitutive equations of the turbulent stress τ^T, the phase change volume generation Γ_g, the pressure distribution p_m and the mixture momentum source $\mathbf{M}_m$. These terms will be taken into account by means of macroscopic laws specifically derived for the 1D model.