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# Transonic flutter analysis using a fully coupled density based solver for inviscid flow

H.I. Kassem*, X. Liu, J.R. Banerjee

*School of Mathematics, Computer Science & Engineering, City University London, Northampton Square, London EC1V0HB, United Kingdom*

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ABSTRACT

This paper focuses on the coupling between the high fidelity aerodynamic model for the flow field and the modal analysis of a typical wing section to carry out flutter analysis. This coupled aeroelastic model is implemented in one of the most widely used open source CFD codes called OpenFOAM. The model is designed to calculate the structural displacement in the time domain based on the free vibration modes of the structure by constructing the numerical model directly from the modal analysis. Essentially a second order ordinary differential equation is solved for each mode as a function of the generalised coordinates. A density based solver using central difference scheme of Kurganov and Tadmor is used to model the flow field. Two main cases of transonic flow over NACA 64A010 are modelled for a forced pitching oscillation airfoil and a self-sustained aerofoil respectively. The self-sustained two degrees of freedom case is modelled for three different possibilities covering damped, neutral and divergent oscillations. Predicted results show very good agreement with the numerical and experimental data available in the literature.



## 1. Introduction

Aeroelasticity is the science of studying the interaction between three main forces namely: elastic, inertia and aerodynamics. Therefore aeroelasticity is an interdisciplinary field combining: fluid mechanics, solid mechanics and structural dynamics. In general, the interaction between these two or three areas is classified as aeroelastic problems. Aeroelastic research started in the late 1920's and the subject matter has matured enormously over the years and now there are many excellent texts on the subject [1–4]. Insufficient or inaccurate prediction of aeroelastic characteristics of aircraft during the design process can lead to catastrophic incidents.

One of the most dangerous aeroelastic instabilities is, of course flutter. It is a self-excited oscillation of elastic body in fluid stream. This condition is usually defined by two important parameters namely the flutter speed and the flutter frequency. Flutter speed defines the speed beyond which the aircraft becomes unstable. It means that if the aircraft flies at this speed it will have steady harmonic oscillation of constant amplitude. This self-excited oscillation will have a frequency which is called the flutter frequency. This point is the most critical point because if for any reason, free stream velocity exceeds the flutter speed, the system will have divergent oscillation and will eventually vibrate in a violent way which could lead to the destruction of the aircraft. The aeroelastic phenomenon, flutter is caused by all three types of forces, namely elastic, inertia and aerodynamics. The fluid flow instead of playing its natural role to damp the structural vibration, it will feed the system instead with more and more energy until divergent oscillation occurs. The complexity of flutter analysis arises from the fact that flutter involves very strong coupling between fluid mechanics and structural dynamics. Therefore an accurate description of the flow field as well as structural dynamic behaviour together with a mechanism of coupling between the two are essential for flutter analysis.

Avoiding flutter is a mandatory requirement in any aircraft design process. Although flutter analysis is a relatively old problem in aviation, it is still challenging, particularly with the advent of composite materials and requirement of high speeds. The main challenge for this problem is at the transonic flow region. The transonic flutter limit appears to be low in any flight range. Therefore for an aircraft the most critical flutter point generally arises when the flow is transonic. The phenomenon is called transonic dip which has featured in the literature many times [5,6]. The transonic flow field is a transition between subsonic flow and supersonic flow exhibiting shock waves and highly non-linear behaviour.

The transonic flow being non-linear poses a great challenge over traditional linear theories [7] which fail to predict accurately the aerodynamic properties. Therefore solving the non-linear governing equations of fluid flow using numerical techniques has become essential [3,8–10]. Despite the computational cost of using

* Corresponding author.
*E-mail address:* hassan.kassem.1@city.ac.uk (H.I. Kassem).

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CFD, it is appropriately being used in the aeroelasticity field for greater accuracy and better flutter prediction. This has given birth to a new field in aeroelasticity called computational aeroelasticity which couples computational fluid dynamics (CFD) with computational structural dynamics (CSD) [11].

In the next section a concise theoretical background is given focusing on the governing equations of the aeroelastic system. Then the numerical methods and the implemented code are explained. Finally, the results of the two validation cases are discussed in detail. This paper is based on an earlier paper [12] but with some enhancement. The essential improvement in this paper appears in the results of the first case study which is improved considerably compared to the previous work. This improvement is mainly due to some refinement in the convergence criteria and better boundary condition for the slip moving wall.

Following the publication of the conference paper by the authors [12], an updated version of the software OpenFOAM-2.3 has now been used [13]. The newer version introduced many improvements, particularly in parallel running performance and the implementation of a new dynamic mesh solver. Also another important improvement in this release is the inclusion of an enhanced ordinary differential equation solver library which is directly relevant to the present work [14]. Due to these modifications some of the implemented features by the authors have been updated in this paper.

## 2. Theoretical background

### 2.1. Aerodynamic model

The governing equations of the flow are the complete Euler equations [15–17]. If $\rho$, $\mathbf{u}$, $p$ and $\boldsymbol{E}$ are density, velocity, pressure and total energy respectively, the Euler equations in vector notation will then have the following form;

- Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\mathbf{u}\rho] = 0 \tag{1}$$

- Conservation of momentum:

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot [\mathbf{u}(\rho \mathbf{u})] + \nabla p = 0 \tag{2}$$

- Conservation of total energy:

$$\frac{\partial (\rho \boldsymbol{E})}{\partial t} + \nabla \cdot [\mathbf{u}(\rho \boldsymbol{E})] + \nabla \cdot [\mathbf{u}p] = 0 \tag{3}$$

where $\nabla$ is the nabla vector operator, $\nabla \equiv \partial_i \equiv \frac{\partial}{\partial x_i} \equiv (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3})$. Thus for any vector $\mathbf{a}$, $\nabla \cdot \mathbf{a}$ is the divergence defined by $\nabla \cdot \mathbf{a} \equiv \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3}$. Also for any scalar $s$, the gradient is $\nabla s \equiv (\frac{\partial s}{\partial x_1}, \frac{\partial s}{\partial x_2}, \frac{\partial s}{\partial x_3})$. In Eq. (3), $\boldsymbol{E} = e + \frac{|\mathbf{u}|^2}{2}$ with $e$ the specific internal energy.

### 2.2. Aeroelastic model

The typical wing section using two-dimensional model [1,3,4] is well established for studying two degrees of freedom wing dynamical system. This model considers the plunging ($h$) and pitching ($\alpha$) motions about the elastic axis of the wing. The governing equations of undamped motion are [18]:

$$m\ddot{h} + S_\alpha \ddot{\alpha} + K_h h = -L \tag{4}$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha \alpha = M_{ea} \tag{5}$$

where $m$, $I_\alpha$ and $S_\alpha$ are aerofoil mass per unit length, polar mass moment of inertia about the elastic axis per unit length and static mass imbalance respectively. $K_h$ and $K_\alpha$ are bending and torsional spring stiffness whereas $L$ and $M_{ea}$ are the lift force (positive up) and moment about the elastic axis (positive nose up). The plunging displacement $h$ is positive down and the angle of attack $\alpha$ is positive nose up and is in radians. Non-dimensionalising the linear displacement by the aerofoil semichord ($b$) in Eqs. (4) and (5) and the time by the uncoupled natural frequency of the torsional spring ($\omega_\alpha$) so that the dimensionless time is $\tau = \omega_\alpha t$. The governing Eqs. (4) and (5) can now be reformulated in the following matrix form

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\} \tag{6}$$

where

$$[M] = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}; \qquad [K] = \begin{bmatrix} (\frac{\omega_h}{\omega_\alpha})^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix} \tag{7}$$

$$\{F\} = \frac{U_\infty^2}{\pi \mu \omega_\alpha^2 b^2} \begin{Bmatrix} -C_l \\ C_m \end{Bmatrix}; \qquad \{q\} = \begin{Bmatrix} \frac{h}{b} \\ \alpha \end{Bmatrix} \tag{8}$$

In Eq. (6), $[M]$ and $[K]$ are the mass and stiffness matrices, and $\{F\}$ and $\{q\}$ are the force and displacement vectors. The non-dimensional aerofoil mass ratio is $\mu = \frac{m}{\pi \rho b^2}$ with $x_\alpha$ and $r_\alpha$ being the static unbalance and the radius of gyration respectively. The uncoupled natural frequencies in plunging and pitching motion are $\omega_h$ and $\omega_\alpha$, respectively. $C_l$ and $C_m$ represent the lift and moment coefficients which have the same sign convention as the aerodynamic forces and moment $L$ and $M$.

### 2.3. Modal analysis

The main objective now is to solve Eq. (6) which represents the aerofoil motion in two degrees of freedom namely the heave and pitch. In order to solve the equations the modal analysis methodology is used. The main concept is representing the system displacements as a linear combination of the free vibration mode shapes through the use of generalised coordinates. In general, if a combination of the first few modes of free vibration say $N$ is used, then according to modal approach the displacement vector can be represented as

$$\{q\} = [\phi]\{\eta\} \tag{9}$$

where $[\phi]$ is the modal matrix in which each column is an eigenvector of the free vibration analysis eigen-problem and $\{\eta\}$ is the generalised coordinates. Premultiplying Eq. (6) by $[\phi]^T$ and substituting using (9) and applying the eigenvectors orthogonality lead to a set of second order ordinary differential equations in generalised coordinates. Each equation is represented by its mode, say $i$th mode [18,19] to give

$$\ddot{\eta}_i + 2\zeta_i \omega_i \dot{\eta} + \omega_i^2 \eta_i = Q_i; \quad i = 1, 2, \ldots, N \tag{10}$$

where

$$Q_i = \{\phi\}_i^T \{F\} \tag{11}$$

$$\omega_i^2 = \{\phi\}_i^T [K] \{\phi\}_i \tag{12}$$

$$1 = \{\phi\}_i^T [M] \{\phi\}_i \tag{13}$$

and $\zeta_i$ in Eq. (10) is modal damping which is not considered in (6). The modes are normalised in a way such that the generalised mass matrix became an identity matrix. In this paper the structural system is considered as an undamped system. However, the damping is shown in Eq. (10) just for reference and showing how the system damping can be considered in the future work.

It is clear from the above equations that to calculate the system displacement vector from Eq. (9), modal matrix $[\phi]$ and the generalised coordinates vector $\{\eta\}$ should be obtained first. Determining