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An equivalent multiscale method for 2D static and dynamic analyses of lattice truss materials

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ABSTRACT

A uniform multiscale computational method is developed for 2D static and dynamic analyses of lattice truss materials in elasticity based on the extended multiscale finite element method. A kind of multi-node coarse element is proposed to describe the more complex deformations compared with the original four-node coarse element and the mode base functions are added into the original multiscale base functions to consider the effects of inertial forces for the dynamic problems. The constructions of the displacement and mode base functions are introduced in detail. In addition, the orthogonality of the displacement and mode base functions are also proved, which indicates that the macroscopic displacement DOF and modal DOF are irrelevant and independent of each other. Finally, some numerical experiments are carried out to verify the validity and efficiency of the proposed method by comparison with the reference solution obtained by the standard finite element method on the fine mesh.

Introduction

Over the past few decades, more and more ultra-light materials are applied to the industrial design because of the advantages such as high stiffness, high strength and light weight. Meanwhile, these materials have also been used in automobile, aerospace and aircraft industries. Among various ultra-light materials, the truss materials have received increasing attentions due to its high stiffness-weight and strength-weight ratios in recent years. The photographs of the 3D lattice truss structures are depicted in Fig. 1 [1,2]. Wallach and Gibson [1] studied the mechanical properties of a particular geometry of 3D truss materials as a function of the aspect ratio of the unit cell. In addition, comparisons of the results between the numerical prediction and experiment have been carried out by Wallach. Deshpande et al. [2] investigated the effective mechanical properties of the octet-truss lattice structure material both experimentally and theoretically. Hutchinson and Fleck [3] applied matrix methods of linear algebra to analyze the structural mechanics of the periodic Kagome lattice and triangular–triangular (T–T) lattice truss based on the Bloch's theorem.

It is unrealistic to solve the large lattice truss structure problems by using the direct numerical method such as finite element method (FEM) because of the limitations of computer memory and computing time. A variety of multi-scale methods and equivalent models are proposed by many researchers to save computational resources for the lattice truss structures. Fan and Yang [4] developed an equivalent continuous method to investigate the effective stiffness of three-dimensional stretching dominated lattice materials. The predicted properties were consistent with the experimental results and this indicated that the proposed continuum model can be used to predict the mechanical properties of the lattice structures. Yan et al. [5] compared the representative volume element (RVE) method based on the homogenization method for predicting the effective elastic property of truss materials with periodic microstructure. Moreau and Caillerie [6] developed a continuum model based on the homogenization method to analyze the geometric large deformation problems of the large periodic beam-like lattice truss structures. Burgardt and Cartraud [7] presented a general procedure based on the energy equivalence to determine the equivalent beam properties of beam-like lattice trusses. Gonella and Ruzzene [8] investigated the equivalent in-plane properties of hexagonal and re-entrant (auxetic) lattices through the analysis of partial differential equations associated with their homogenized continuum models. The general static and dynamic mechanical properties of the lattice structures are estimated by the presented continuum model based on the homogenization technique. Furthermore, Gonella and Ruzzene [9] also proposed a multiscale formulation for one-dimensional spring-mass system and two-dimensional truss lattice structure based on the homogenization

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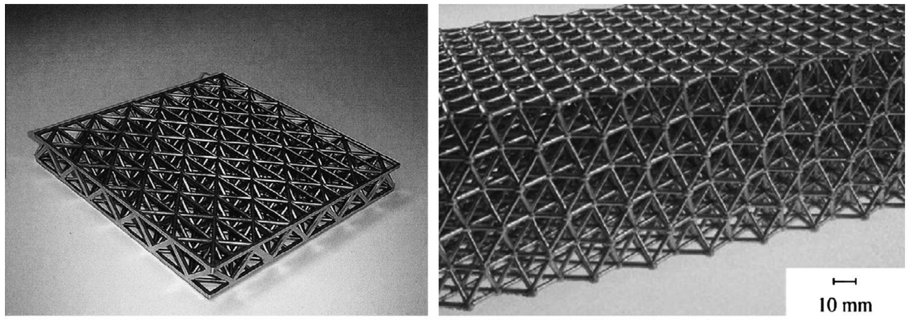

(a) (b)

Fig. 1. Photographs of the lattice truss structures [1,2].

technique and applied this formulation to investigate the elastodynamic properties of the periodic lattice truss structures. Vigliotti and Pasini [10] presented a multiscale procedure for the linear analysis of the structure with lattice material. Elsayed and Pasini [11] discussed the design of the microstructure of a lattice material with regular octet-truss cell topology and the multiscale design of an axially loaded member manufactured of this type of cellular solid. Zeman et al. [12] studied the mechanical behaviors of statistically non-uniform two-phase elastic discrete lattice structures based on the methodology proposed by Luciano and Willis [13]. Vigliotti and Pasini [14] recently presented a multiscale procedure for the linear analysis of the stiffness and strength of open and closed three-dimensional cell lattices with arbitrary topology.

Similarly, several other multiscale methods which were developed to save the computing resources are presented recently. Fish and Yuan proposed a multiscale enrichment method based on the partition of unity (MEPU) [15] and then extended this method to the nonperiodic fields and nonlinear problems successfully [16]. By employing the generalized variational principles, Xu et al. [17] developed a Green-function-based multiscale method to decompose the boundary value problem with random microstructure into a slow scale deterministic problem and a fast scale stochastic one. Then, Shen and Xu [18] discussed the computational aspects of this method in detail. Liu and McVeigh [19] derived a general multiscale theory for modeling heterogeneous materials via a nested domain based virtual power decomposition. Lim et al. [20] proposed a kind of variable-node finite element with smoothed integration for the multiscale mechanics problems.

The multiscale finite element method (MsFEM), which can be traced back to the work presented by Babuska et al. [21,22], has been used by Hou et al. [23,24] to solve the boundary value problems with high oscillating coefficients. This multiscale method has been widely used to simulate the multiscale problems [25,26]. To study the problems in solid mechanics, Zhang et al. [27] developed a coupling multiscale finite element method (CMsFEM) for consolidation analysis of heterogeneous saturated porous media. Furthermore, an extended multiscale finite element method (EMsFEM) was proposed by Zhang et al. for the elastic and elasto-plastic static analysis of periodic lattice truss materials [28,29]. In EMsFEM, the multiscale base functions are numerically constructed and can capture the heterogeneous properties at the microscopic level effectively. Particularly, the additional coupling terms of the multiscale numerical base functions among different directions in multi-dimensional problems are taken into consideration, which can improve the computational accuracy significantly.

In this paper, a uniform multiscale computational method is developed based on EMsFEM for 2D static and dynamic analyses of the lattice truss materials on elasticity. As we all know, for the static problems, the displacement of the structure is associated with the stiffness of the structure and the external force directly. While for the dynamic problems, the inertial force of the structure also needs to be considered. Therefore, the multiscale numerical base functions in EMsFEM should reflect the effect of the inertial force of the structure for the dynamic analysis. Just because of this, the mode base functions are introduced into the original multiscale base functions to improve the computational accuracy for the dynamic problems. On the other hand, the coarse element used in EMsFEM is usually the four-node coarse element which can only accurately describe the simple deformations, and this four-node coarse element would be powerless for the more complex deformations in the dynamic analysis. In this context, one will have to refine the coarse-scale meshes when calculating the 2D dynamic problems with the four-node coarse element, and this will reduce greatly the computational efficiency and advantages of EMsFEM. Therefore, in order to ensure the computational accuracy without increasing too much amount of computational cost, it is necessary to modify the four-node coarse element to describe the more complex deformations. For this reason, a kind of multi-node coarse element is proposed in this paper.

The research significances of this multiscale method in the practical engineering are very clear. For the large and complex truss structures, the traditional numerical methods in current commercial software can only simulate their mechanical behaviors by refining the mesh to the size of a single truss element. This is bound to spend huge computer memory and CPU time. Moreover, for some extra large problems, the current commercial software even cannot obtain a valid solution under the present circumstances due to the restrictions of the computer memory and computation time. By comparison, this multiscale method can not only obtain a relatively accurate result, but also reduce a large amount of computer memory and computing time. Therefore, it has practical engineering significances to study this multiscale method.

The contents of this paper are as follows. In Section 'Brief review of the extended multiscale finite element method (EMsFEM)', a brief review of the extended multiscale finite element method (EMsFEM) is given. Then, some fundamental principles of the proposed multiscale computational method are introduced in Section 'Basic principles of the uniform multiscale computational method'. The static analysis, generalized eigenvalue analysis and dynamic response analysis are presented in Sections 'Static analysis', 'Generalized eigenvalue analysis' and 'Dynamic response analysis', respectively. Finally, some conclusions are given in Section 'Conclusions'.

Brief review of the extended multiscale finite element method (EMsFEM)

The implementation of the EMsFEM mainly consists of two parts [30]. One is the microscopic computation in which the multiscale base functions of each coarse element are constructed independently by solving the local static equilibrium equations