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# Finite element model for damping optimization of viscoelastic sandwich structures

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ABSTRACT

In this work a simple and efficient finite element model is used for the damping optimization of multilayer sandwich plates, with a viscoelastic core sandwiched between elastic layers, including piezoelectric layers. The elastic layers are modeled using the classical plate theory and the core is modeled using Reddy's third-order shear deformation theory. The finite element formulation is obtained by assembly of N "elements" through the thickness, using specific assumptions on the displacement continuity at the interfaces between layers. The free vibration response of damped multilayer sandwich structures is characterized by solving an eigenvalue problem to obtain the fundamental natural frequency and corresponding modal loss factor. The optimization is conducted in order to maximize the fundamental modal loss factor, using gradient based algorithms, and afterwards, considering steady state harmonic motion the analysis is conducted in time domain to obtain the structure response. The model is applied in the solution of some illustrative examples and the results are presented and discussed.



## 1. Introduction

Viscoelastic sandwich composites are structures in which a viscoelastic layer is sandwiched between elastic layers, and are widely used in engineering applications in order to reduce vibration amplitude and noise. In this situation, the passive damping is introduced by the strong transverse shear in the core. Using simultaneously viscoelastic and piezoelectric layers, we have a hybrid structure with an active–passive damping treatment. That is, the piezoelectric actuator uses the active control mechanism based on induced in-plane piezoelectric actuation strains, and the passive constrained layer uses its passive damping mechanism based on vibratory energy dissipation through transverse shear strains induced in the viscoelastic layer [1].

Initially, analytical models were developed to obtain approximate loss factors and natural frequencies of sandwich beams or plates with viscoelastic core, with simply supported boundary conditions. These can be found in the works of DiTaranto and Blasingame [2], Mead and Markus [3], Yan and Dowell [4], and Rao [5], among others.

The use of finite element models for the analysis of damped beams and plate sandwich structures, is found in some works in the literature. Rikards et al. [6] present laminated superelements formed through simple beam or plate finite elements for each layer, with four nodes or six nodes for beam or plate respectively, and based on the first-order shear deformation theory. Yi and Hilton [7] developed a rectangular plate element model based on Mindlin theory. Jonhson et al. [8] and Lu et al. [9] present a model obtained by two plate elements for the face layers and one solid element for the viscoelastic core. Moreira et al. [10] developed a model with a 4-node facet type quadrangular shell finite element, based on layerwise theory, and benchmarked it on the analysis of damped beams and plate sandwich structures.

Finite element models for active–passive structures, have been proposed among others by Boudaoud et al. [11] which presented a five-layered finite element for control of composite structures with piezoelectric and viscoelastic layers, and recently Araújo et al. [12] developed an eight node serendipity sandwich plate finite element formulated using a mixed layerwise approach, where the assumed displacement field for the viscoelastic core is based on a third order expansion of the thickness coordinate, regarding in-plane displacements and a first order theory for the elastic and piezoelectric layers.

Optimal design of constrained layer damping treatments of vibrating structures focused at the maximization of modal damping ratios is also an important research field. Baz and Ro [13] optimized constrained layer damping treatments considering the thickness and shear modulus of the viscoelastic layer as design variables. Lifshitz and Leibowitz [14] also using the layer thicknesses as design variables, determined the optimal passive constrained layer damping. Araujo et al. [15,16] developed a mixed layerwise sandwich finite element model for analysis and damping optimization of viscoelastic laminated sandwich composite structures.

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In this work we present optimization of the fundamental modal loss factor of sandwich structures using gradient based algorithms, using a simple and efficient finite element model, developed by Moita et al. [17], and non-linear mathematical programming techniques described by Vanderplaats [18]. Thus, the modal loss factor is the objective function and the design variables are the fiber angles of elastic layers, and the thicknesses of the elastic and viscoelastic layers. The present model is applied to beam, plate and a shell structure and results are compared with alternative solutions.

## 2. Sandwich plate model

Fig. 1 shows a typical sandwich plate for analysis of hybrid sandwich laminated plates with a viscoelastic core (v), laminated anisotropic face layers (e1, e2) and piezoelectric sensor (s) and actuator (a) layers.

Sandwich plates with viscoelastic cores are very effective in reducing and controlling vibration response of structures. Due to the high shear developed inside the core, and to the high ratios of skin to core stiffness, equivalent single layer plate theories are not adequate to describe the behavior of these structures. The finite element model that is used in this work to carry out the optimization process is obtained by assembly of N "elements" throughout the thickness, enforcing displacement continuity at the interfaces between layers [17].

Although sensor and actuator layers are included in the formulation presented here, the piezoelectric effect will not be considered for optimization purposes in this work, as it deals only with optimization of passive damping.

For the viscoelastic layer, Reddy's third-order shear deformation theory [19] is assumed. Thus, the displacement field is:

$$\begin{aligned} u(x,y,z,t) &= u_0^v(x,y,t) - z\theta_y(x,y,t) + z^3 c_1\left[\theta_y(x,y,t) - \tfrac{\partial w_0}{\partial x}\right] \\ v(x,y,z,t) &= v_0^v(x,y,t) + z\theta_x(x,y,t) + z^3 c_1\left[-\theta_x(x,y,t) - \tfrac{\partial w_0}{\partial y}\right] \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (1)$$

where $u_0^v, v_0^v, w$ are displacements of a generic point in the middle plane of the core layer referred to the local axes – $x, y, z$ directions, $\theta_x, \theta_y$ are the rotations of the normal to the middle plane, about the $x$ axis (clockwise) and $y$ axis (anticlockwise), $\partial w_0/\partial x$, $\partial w_0/\partial y$ are the slopes of the tangents of the deformed mid-surface in $x$, $y$ directions, and $c_1 = 4/3h^2$, with $h$ denoting the total thickness of the structure.

For the elastic and piezoelectric layers the Kirchhoff–Love theory is considered. The corresponding displacement field is:

$$\begin{aligned} u^i(x,y,z,t) &= u_0^i(x,y,t) - (z - z_i)\tfrac{\partial w_0}{\partial x} \\ v^i(x,y,z,t) &= v_0^i(x,y,t) + (z - z_i)\left(-\tfrac{\partial w_0}{\partial y}\right) \\ w^i(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (2)$$

where $u_0^i$, $v_0^i$ are the in-plane displacements of a generic point in the middle plane of the $i$ layer, $\partial w_0/\partial x$, $-\partial w_0/\partial y$ are the slopes of the tangents of the deformed mid-surface in $x$, $y$ directions respectively, and $z_i$ is the $z$ coordinate of the mid-plane of each layer, with reference to the core layer mid-plane, and $i = s, e_1, e_2, a$ is the index of sensor, upper and lower elastic, and actuator layers respectively.

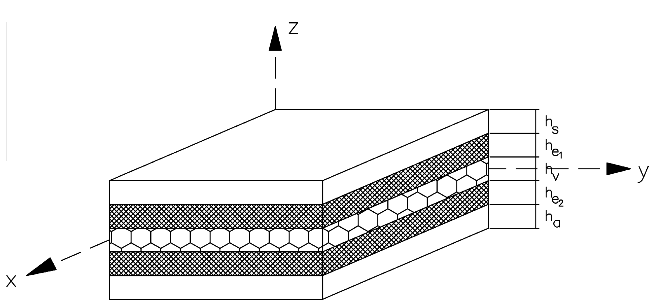

**Fig. 1.** Sandwich plate.

In this formulation, an exact continuity between layers is considered. Thus, the displacement field in any layer can be obtained from the displacement field of the viscoelastic layer, taking into consideration the conditions of kinematic links, as shown in Moita et al. [17]. Hence, the number of degrees-of-freedom (DOF) is reduced to 7 mechanical nodal DOF: $u_0^v, v_0^v, w_0, -\partial w_0/\partial y, \partial w_0/\partial x, \theta_x$ and $\theta_y$. A linear electric potential field through the thickness direction of each piezoelectric layer is also assumed, leading to 1 electrical DOF per piezoelectric layer.

It is assumed here, that the laminate consists of several layers, including the piezoelectric, elastic, and viscoelastic layers. The constitutive equation for the last two types of layers, are

$$\bar{\boldsymbol{\sigma}} = \bar{\mathbf{Q}}\bar{\boldsymbol{\varepsilon}} \quad (3)$$

and the constitutive equations of a deformable piezoelectric material, coupling the elastic and the electric fields are given by [20].

$$\begin{aligned} \bar{\boldsymbol{\sigma}} &= \bar{\mathbf{Q}}\bar{\boldsymbol{\varepsilon}} - \bar{\mathbf{e}}\bar{\mathbf{E}} \\ \bar{\mathbf{D}} &= \bar{\mathbf{e}}^T\bar{\boldsymbol{\varepsilon}} + \bar{\mathbf{p}}\bar{\mathbf{E}} \end{aligned} \quad (4)$$

where all the quantities appearing in the above equations are explicitly defined in [17].

For the case of viscoelastic materials, the engineering constants are taken to be complex:

$$\begin{aligned} E_1 &= E_1'(1 + i\eta_{E_1}) \\ E_2 &= E_2'(1 + i\eta_{E_2}) \\ \nu_{12} &= \nu_{12}'(1 + i\eta_{\nu_{12}}) \\ G_{12} &= G_{12}'(1 + i\eta_{G_{12}}) \\ G_{13} &= G_{13}'(1 + i\eta_{G_{13}}) \\ G_{23} &= G_{23}'(1 + i\eta_{G_{23}}) \end{aligned} \quad (5)$$

where the prime (′) quantities are storage modulus, $\eta$ denotes material loss factors and $i = \sqrt{-1}$. It should be noted that in Eq. (5) both storage modulus and loss factors are, in general, frequency dependent.

The electric field vector is the negative gradient of the electric potential $\phi$, which is assumed to be applied in the thickness direction, where it can vary linearly, i.e.

$$\bar{E} = -\nabla\phi = \{0 \quad 0 \quad E_z\}^T; \quad E_z = -\phi/h_p \quad (6)$$

where $h_p$ is the thickness of a piezoelectric layer.

## 3. Finite element formulation

A non-conforming triangular finite element model is used with three nodes and seven degrees of freedom per node, the displacements $u_{0_i}, v_{0_i}, w_{0_i}$, the slopes $(-\partial w_0/\partial y)_i, (\partial w_0/\partial x)_i$, and the rotations $\theta_{xi}, \theta_{y}i$. To solve shell problems we consider fictitious stiffness coefficients $K_{\theta z}$, to eliminate the problem of a singular stiffness matrix, when the elements are coplanar, and an extra degree of freedom of freedom $\theta_{zi}$ is introduced per node [17,21,22].

The element local displacements, rotations and slopes, are expressed in terms of nodal variables through shape functions $N_i$ given in terms of area co-ordinates $L_i$. The displacement and strain fields are explicitly defined in [17].

### 3.1. Free vibration analysis

The dynamic equations of a sandwich plate, including piezoelectric layers, can be derived from the Hamilton's principle. The