



A practical test for noisy chaotic dynamics

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Received 12 April 2015; received in revised form 15 June 2015; accepted 21 August 2015

Abstract

This code computes the largest Lyapunov exponent and tests for the presence of a chaotic dynamics, as opposed to stochastic dynamics, in a noisy scalar series. The program runs under MATLAB[®] programming language.

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Keywords: Lyapunov exponent; Noisy chaos; MATLAB

Code metadata

Current code version	v5.0
Permanent link to code/repository used for this code version	https://github.com/ElsevierSoftwareX/SOFTX-D-15-00007
Legal Code License	BSD-3-Clause, see http://opensource.org/licenses/BSD-3-Clause
Code versioning system used	none
Software code language	MATLAB [®] [1]
Requirements	statistics and optimization toolboxes
Support email for questions	ahmedbensaida@yahoo.com

1. Motivation and significance

In a scientific context, the word chaos has a different meaning than it does in its general usage as a state of confusion or a total disaster. Chaos, with reference to chaos theory, refers to an apparent lack of order in a system that nevertheless obeys particular laws or rules, a condition discovered by the physicist Henri Poincaré in the early 20th century, which refers to an inherent lack of predictability in some physical systems known as dynamical instability. For example, chaos is observed in the movement of a driven pendulum where it may behave erratically and show irregular sequences of left and right turns, streams in the ocean [2,3], and in meteorological science [4,5]. Hence, a chaotic system – no matter how complex it may be – relies upon a precise order, and very small changes can cause

very complex behaviors, known as *sensitive dependence on initial conditions*. This random-like and unpredictable behavior is known as *the butterfly effect*.

The butterfly effect, first described by Edward Lorenz at the December 1972 meeting of the American association for the advancement of science in Washington D.C., vividly illustrates the essential idea of chaos theory. In nonlinear dynamics theory, a butterfly flapping its wings in Brazil can produce a tornado in Texas due to nonlinearities in weather processes. The example of such a small system, as a butterfly, being responsible for creating such a large and distant outcome, as a tornado in Texas, illustrates the impossibility of making predictions for complex systems despite the fact that these are determined by underlying conditions that can never be sufficiently articulated to allow long-range predictions.

Chaotic dynamics are closely related in appearance to stochastic dynamics, and the BDS test [6] cannot separate them [7]. Hence, we need a practical test to detect chaos even when the data are noisy.

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Theoretical basis of the code are available in [7,8], practical use of the test is performed by [9,10], among others. The reminder of the article is as follows: in Section 2 we briefly describe the code; Section 3 presents some examples; in Section 4 we discuss the impact of the code; and finally, we conclude in Section 5. Code metadata is given before Section 1.

2. Software description

Tests for chaos are scarce in the literature, and practical implementation is far from evident [11]. Moreover, to test for chaotic dynamics, we need to have pure data free from frictions engendered by measurement errors, see [12] for detailed discussion. The present code estimates the *largest Lyapunov exponent* (Lyapunov exponent henceforth) in a noisy time series [7], and decides whether the data are chaotic or stochastic based on a confidence level α . The main advantage of this test is that it can be conducted directly on experimental data without the need to define the generating equations.

The Lyapunov exponent λ measures the average exponential divergence (positive exponent) or convergence (negative exponent) rate between nearby trajectories within a time horizon that differ in initial conditions only by an infinitesimally small amount. We distinguish 3 cases of λ :

- $\lambda < 0$: the orbit attracts to a stable fixed point. A negative Lyapunov exponent is characteristic of dissipative or non-conservative system. Such a system exhibits asymptotic stability; the more negative the exponent, the greater the stability. Super-stable fixed points and super-stable periodic points have $\lambda \rightarrow -\infty$.
- $\lambda = 0$: the orbit is a neutral fixed point. A Lyapunov exponent of zero indicates that the system is in a steady-state mode or near the transition to chaos.
- $\lambda > 0$: the orbit is unstable and chaotic. Nearby points of an orbit, no matter how close, will diverge to any arbitrary separation. The larger the exponent, the more unstable the system.

To compute the Lyapunov exponent and to conduct the test given the hidden noise in the data, we need to define two components:

1. The activation function `ActiveFN` of the neural network that approximates the chaotic map [7].
2. The order (L, m, q) that defines the complexity of the neural network [7]. Choosing low parameters may prevent the neural network from reasonably approximating the map that generates the data. On the other hand, large parameters increase computational time exponentially because the number of coefficients to estimate increases. [8] suggests any triplet between $(5, 6, 5)$ and $(10, 12, 10)$.

2.1. Software architecture

The code is a single m -file that runs under MATLAB® programming language [1], all subroutines are included in the principal file `chaostest`.¹

¹ We are planning to write a version that runs under R, <http://CRAN.R-project.org>.

2.2. Software functionalities

The code `chaostest` can detect the presence of chaotic dynamics. It tests the positivity of the dominant (or largest) Lyapunov exponent λ at a specified confidence level.

The test hypothesis H are: null hypothesis $H_0 : \lambda \geq 0$, which indicates the presence of chaos; and alternative hypothesis $H_1 : \lambda < 0$, which indicates the absence of chaos.

The syntax under MATLAB® command prompt is:



```
Command Window
>> [H, pVal, LAMBDA, Orders, CI] = chaostest(Series, ActiveFN, maxOrders, ALPHA)
```

Required input

`Series`—a vector of observations to test. The minimum allowed size is $(mL + 1)$ observations.

Optional inputs

`ActiveFN`—string containing the activation function to use in the neural network estimation. It can be:

'tanh', $f(u) = \tanh u$, domain = $(-1, 1)$, this is the default;

'logistic', $f(u) = \frac{1}{1+e^{-u}}$, domain = $(0, 1)$. The logistic is not recommended due to the limited domain;

'sigmoid', $f(u) = u \frac{1+|u|}{2+|u|+\frac{u^2}{2}}$, domain = $(-1, 1)$.

`maxOrders`—the orders (L, m, q) . This must be a vector containing 3 elements, default = $[5, 6, 5]$.

`ALPHA`—the significance level α of the test (default = 0.05).

Outputs

`H = 0`—accept the null hypothesis of chaos at significance level α .

`H = 1`—reject the null hypothesis of chaos at significance level α .

`pVal`—the p -value. Small values of `pVal` cast doubt on the validity of the null hypothesis of chaos.

`LAMBDA`—the Lyapunov exponent λ .

`Orders`—gives the triplet (L, m, q) that maximizes the Lyapunov exponent computed from all $L \times m \times q$ estimations, see [7] for further details.

`CI`—confidence interval for λ at significance level α .

The algorithm uses the Jacobian method [7], it needs the optimization and the statistics toolboxes of MATLAB® [1].

3. Illustrative examples

In this section we perform three examples. The first one for a noisy chaotic map, the second one for a stochastic random variable, and the last one for a real-life data.

3.1. The logistic map

The logistic map $x_t = \gamma x_{t-1} (1 - x_{t-1})$ is known to exhibit chaotic behavior when $3.57 \leq \gamma \leq 4$ [11]. So, in our example we set $\gamma = 4$ and a starting value $x_0 = 0.2$. Next, to the constructed pure chaotic map x_t , we add some noise ε_t , with $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is an independently and normally distributed random variable.

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