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A model and topological analysis procedures for a pipeline network of variable connectivity

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ABSTRACT

In the paper, a model is firstly formulated for a pipeline network of variable connectivity in the terminology of graph theory. Through analyzing the topological changes of the pipeline network caused by connecting or disconnecting a pipe, several procedures are then proposed to construct the incidence matrix and fundamental circuit matrix of a graph directly from those of its parent graph or graphs without the time-consuming inversion of the corresponding incidence matrix. Thirdly, the proposed model and topological analysis procedures are used to establish a dynamic solver for a tank farm together with the chord flow method of Rahal (1995) [3]. Finally, the dynamic solver is applied to a tank farm of liquor for verifying the model and procedures proposed in this paper.

1. Introduction

Pipeline networks are the major facilities of water, air and heating supply systems. They are also important integral parts of large-scale chemical, petrochemical, petroleum refining and processing plants and various tank farms. Their safe and efficient operation has a great significance for both themselves and other pieces of equipment connected by them. For design and operation studies with simulation, many efforts such as Martinez-Benet and Puigjaner [1], Nielsen [2], Rahal [3], Álvarez et al. [4], and Ivanov and Bournaski [5], have been devoted to the development of steady state and dynamic mathematical models for municipal water distribution networks. To get the pressure and flow dynamics in a pipeline network, the pseudo-steady state models for slow transients, the water hammer models for rapid transients, or models hybrid of the above two kinds are employed. These models involve sets of rigid differential equations which are usually difficult to be solved.

In modeling the dynamics of chemical processes or tank farms, pipeline networks are different from those in water distribution systems: (1) the scale of networks is usually much smaller; (2) the dynamics of networks is neglected since the overall dynamics is usually governed by equipment units other than the networks themselves. Therefore, a static solver for a pipeline network can be used as the dynamic simulator for the corresponding chemical process or tank farm with the pressures at nodes corresponding to tanks or other liquid accumulators changing with time and operations according to the level equations. In composing the static solver of this paper, the chord flow method of Rahal will be employed for its high execution speed, low demand for memory, and insensitivity to initial values.

Such a static or dynamic solver works only for a pipeline network or sub-network which can be expressed by a connected graph or subgraph. In dynamic simulation, however, the connectivity of a network may be changed due to connecting and disconnecting pipes by the operations of their affiliated valves, switches or pumps. A network may split into sub-networks with smaller scales, and sub-networks may split further or merge into another sub-network with a larger scale. That is, the overall graphical topology of a pipeline network changes, and more than one connected subgraphs are produced. To apply Rahal's method to such a pipeline network, topological analysis is necessary to identify the involved connected sub-networks and the corresponding incidence matrices and fundamental circuit matrices.

In this paper, we use the following strategy to analyze the topology of a pipeline network of variable connectivity: (1) operations of connecting or disconnecting pipes are treated one by one. (2) For each operation, procedures are suggested to derive the incidence matrix and the fundamental circuit matrix of a graph or subgraph directly from those of its parent subgraphs or graph, without the time-consuming inversion of the incidence matrix for determining the fundamental circuits.

The rest of this paper is organized as follows: in Section 2, a general framework is constructed for modeling a pipeline network of variable connectivity. In Section 3, procedures are proposed for

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analyzing the topological changes of a pipeline network caused by the connection or disconnection of a pipe. In Section 4, a static solver based on the chord flow method of Rahal is detailed for a connected sub-network. Based the topological analysis procedures and the static solver, a dynamic solver is developed for a tank farm in Section 5. Section 6 is a demonstration for verifying the proposed modeling framework, procedures for topological analysis, and dynamic solver. Conclusions are drawn in Section 7.

2. Model for a pipeline network of variable connectivity

By a pipeline network of variable connectivity, we mean primarily a pipeline network of which the connections of the pipes can be changed by connecting and disconnecting pipes through the operations of their affiliated valves, switches or pumps, though connectivity also is a measure of the fluid-passing capability of a pipe related to the opening degree(s) of some valve(s). Obviously, the connectivity of such a network is certain at each instant of time. Therefore, such a pipeline network is also referred to as a pipeline network or simply a network for simplicity, when it is studied at some certain time instant.

A pipeline network with or without variable connectivity can be viewed as a directed graph with pipes being the arcs and junctions of pipes being the nodes. A junction can be a joint at which several pipes jointed together or a liquid tank, an import source of liquid, or an export destination. Nodes for normal pipe joints are called fixed demand nodes, whereas tanks, sources and destinations are represented by fixed head nodes with constant pressures or pressures specified outside the network. While an arc can be defined to represent a real pipe (therefore, a real arc hereafter), we also have pseudo-arcs in order to follow the method of Rahal. According to Rahal, for a graph with r fixed head nodes, $(r-1)$ pseudo-arcs are introduced to link one fixed head node (the reference node) to the other $(r-1)$ ones. In this paper, the network in question is assumed to have $b \geqslant 0$ pipes and $n > 0$ junctions among which $0 \leqslant r \leqslant n$ junctions represent the involved tanks, sources and destinations. Therefore, we have a directed graph (G) of n nodes (N_i, $i = 1,2,\ldots,n$) and $(b + r - 1)$ arcs ($B_k, k = 1,2,\ldots,b + r - 1$). Furthermore, the algorithm of this paper requires that the r fixed head nodes be indexed after the $(n - r)$ fixed demand nodes, that is, N_i is a fixed demand node for $i = n - r + 1,\ n - r + 2,\ldots,n$. Similarly, the $(r - 1)$ pseudo-arcs are indexed after the b real arcs, namely, B_i is a pseudo-arc for $i = b + 1,\ b + 2,\ldots,b + r - 1$.

2.1. Node and its model

As shown in Fig. 1, a node (N_i) is an airtight space of zero volume (a point) to join arcs ($B_k, k = I_1, I_2,\ldots,I_u$) together, has a pressure of p_i and a height of h_i meters above the reference plane, and is demanded to discharge fluid at a flowrate of d_i (negative value of d_i for liquid entrance). For this node, the following mass balance equation holds

$$\sum_{k=I_1}^{I_u} Q_k - d_i = 0 \tag{1}$$

where Q_k is the flowrate of fluid passing through arc k.

If node i represents a liquid tank, its pressure (at the bottom of the tank) can be calculated by the following equation

$$p_i = p_i^0 + \rho g H_i \tag{2}$$

where ρ is the liquid density, p_i^0 the pressure over the liquid in the tank, and H_i the liquid level determined by

$$\frac{dH_i}{dt} = -\frac{1}{S_i}\left(\sum_{k=I_1}^{I_u} Q_k + d_i\right) \tag{3}$$

with S_i being the cross section area of the tank. In this paper, we assume that the tank represented by node i has a bottom h_i meters high above the reference plane, and all the pieline linked to the tank are connected at the tank's bottom. For node i representing an import source or an export destination, a boundary condition for the network in question, we assume that the node also is h_i meters high above the reference plane and has a pressure p_i specified outside the network.

It is noted that in this paper, we assume that the discharge flowrates of all the fixed demand nodes be zero ($d_i = 0$). For dyanamic simulation of a chemical process or a tank farm, it is realistic and more convenient to take the boundary of a pipeline network as fixed head nodes.

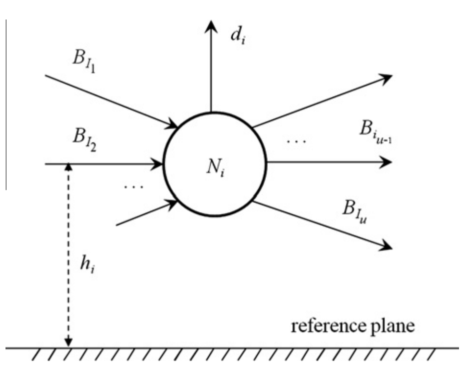

Fig. 1. A node N_i.

2.2. Arc and it model

As shown in Fig. 2, an arc (B_k) is an airtight channel for fluid flow from node N_{k0} to N_{k1}, and is characterized by an element E_k which may be a simple pipe ($E_k = 1$), a valve ($E_k = 2$), a pump ($E_k = 3$), or a pseudo-arc ($E_k = 4$) as seen later. The flowrate through B_k is Q_k, and a negative value of Q_k indicates a flow from node N_{k1} to N_{k0}. The model of an arc is stated below for different arc elements, respectively.

2.2.1. Model of a simple pipe ($E_k = 1$)

On a simple pipe, only a two-position switch (O_k) is mounted to make the pipe fully open ($O_k = 1$) or completely shutoff ($O_k = 0$). For a pipe without such a switch, simply set $O_k = 1$. From the fundamental of fluid dynamics [6], we have the following pressure drop vs. flowrate model for this simple pipe at $O_k = 1$:

$$\Delta P_k = p_{k0} - p_{k1} = \delta_k Q_k - \rho g (h_{k0} - h_{k1}) \tag{4}$$

where ΔP_k is pressure difference, and δ_k the pressure-drop coefficient calculated by

$$\delta_k = f_{pk} \frac{l_k}{d_k} \frac{\rho}{2} |Q_k| \tag{5}$$

with l_k being the pipe's length, d_k the pipe's diameter, and f_{pk} the Darcy friction factor evaluated by the formula of Churchill [7] for all the three (laminar, transitional and turbulent) flow regions

$$f_{pk} = 8\left[\left(\frac{8}{R_e}\right)^{12} + (R_1 + R_2)^{-3/2}\right]^{1/12} \tag{6}$$

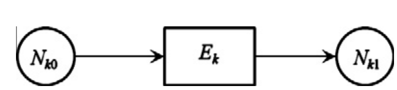

Fig. 2. An arc B_k.