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Total FETI based algorithm for contact problems with additional non-linearities

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ABSTRACT

The paper is concerned with application of a new variant of the FETI domain decomposition method called Total FETI to the solution to contact problems. Its basic idea is that both the compatibility between adjacent sub-domains and Dirichlet boundary conditions are enforced by Lagrange multipliers. We introduce the Total FETI technique for solution to the variational inequalities governing the equilibrium of system of bodies in contact. Moreover, we show implementation of the method into a code which treats the material and geometric non-linear effects. Numerical experiments were carried out with our in-house general purpose finite element package PMD.

1. Introduction

Modelling contact phenomena is still a challenging problem of non-linear computational mechanics. The complexity of such problems arises from the fact that we do not know the regions in contact until we have run the problem. Their evaluations have to be part of the solution. In addition, the solution across the contact interface is non-smooth.

In 1991 Farhat and Roux came up with a novel domain decomposition method called FETI (Finite Element Tearing and Interconnecting) [1]. This method belongs to the class of non-overlapping totally disconnected spatial decompositions. Its key concept stems from the idea that satisfaction of the compatibility between spatial sub-domains, into which a domain is partitioned, is ensured by the Lagrange multipliers, or forces in this context. After eliminating the primal variables, which are displacements in the displacement based analysis, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is solved iteratively. The CPU time that is necessary for both the elimination and iterations can be reduced nearly proportionally to the number of processors, so that the algorithm exhibits the parallel scalability. This method has proved to be one of the most successful algorithms for parallel solution of problems governed by elliptic partial differential equations. Observing that the equality constraints may be used to define so called 'natural coarse grid', Farhat, Mandel and Roux modified the basic FETI algorithm so that they were able to prove its numerical scalability, i.e. asymptotically linear complexity [2].

The fact that sub-domains act on each other in terms of forces suggests that the FETI approach can also be naturally applied to solution to the contact problems with great benefit. To this effect the FETI methodology is used to prescribe conditions of non-penetration between bodies. We shall obtain a new minimisation problem with additional non-negativity constraints which replace more complex general non-penetration conditions [3]. It turned out that the scalability of the FETI methods may be preserved even for solution to the contact problems [3,4].

A new variant of the FETI method, called the Total FETI (TFETI) method was presented in [5]. In this paper, we are concerned with application of this method to solution to the contact problems while we in addition consider the material and geometric non-linear effects.

We briefly introduce theoretical foundations of the FETI and TFETI methods. Then we describe an algorithm, in which the TFETI based contact solver accounts for the inner loop, while the outer loop is concerned with the non-linear effects others than the contact. Numerical experiments were carried out with our in-house general purpose finite element package PMD (Package for Machine Design) [6].

2. The primal problem

Let us consider the static case of a contact problem between two solid deformable bodies. This is basically the boundary value problem known from the continuum solid mechanics. The problem is depicted in Fig. 1. Two bodies are denoted by $(\Omega_1, \Omega_2) \subset \mathbf{R}^n, n = 2$

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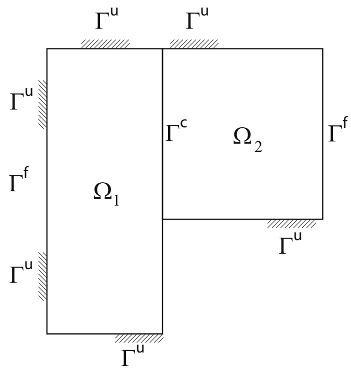

Fig. 1. Basic notation of the contact problem.

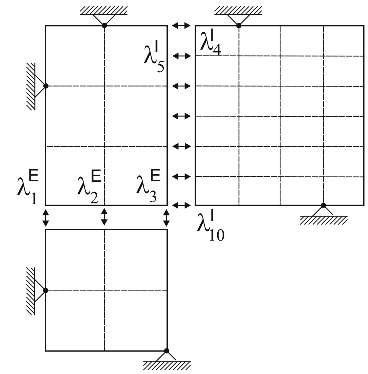

Fig. 3. Principle of the FETI method.

or $n = 3$, where n stands for number of Euclidean space dimensions. Γ denotes their boundary. We assume that the boundary is subdivided into three disjoint parts. The Dirichlet and Neumann boundary conditions are prescribed on the parts Γ^u and Γ^f, respectively. The third kind of the boundary, Γ^c, is defined along the regions where contact occurs and can in general be treated as both the Dirichlet or Neumann conditions. The governing equations are given by the equilibrium conditions of the system of bodies. In addition to these equations, the problem also is subject to the boundary conditions; see, e.g., [7, Chapter 2] for comprehensive survey of formulations.

Fig. 2 shows the discretised version of the problem from Fig. 1. Both sub-domains are discretised in terms of the finite elements method. This figure also shows applied Dirichlet boundary conditions, some displacements, denoted as u, associated with the nodal points, and the contact interface. The displacements are the primal variables in the context of the displacement based finite element analysis.

3. The original FETI method

The result of application of the FETI method to the computational model from Fig. 2 is depicted in Fig. 3. The sub-domain Ω_1 is decomposed into two sub-domains in this case with fictitious interface between them. The contact interface remains the same. The fundamental idea of the FETI method is that the compatibility between sub-domains is ensured by means of the Lagrange multipliers or forces. λ^E denotes the forces along the fictitious interface and λ^I stands for the forces generated by contact.

Let N be a number of sub-domains and let us denote for $i = 1, \ldots, N$ by K_i, f_i, u_i and B_i the stiffness matrix, the vector of externally applied forces, the vector of displacements and the

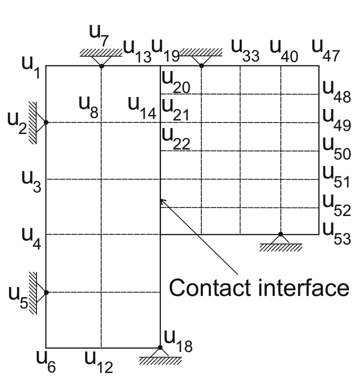

Fig. 2. The primal problem.

signed matrix with entries $-1, 0, 1$ defining the sub-domain interconnectivity for the ith sub-domain, respectively. The matrix B is composed of matrices B^I and B^E, $B = [B^I B^E]$. B^E introduces connectivity conditions along the fictitious interfaces and B^I along the contact ones.

The discretised version of the problem is governed by the following quadratic form

$$\min \quad \frac{1}{2} u^\top K u - f^\top u \quad \text{s.t.} \quad B^I u \leqslant 0 \quad \text{and} \quad B^E u = 0 \tag{1}$$

where

$$K = \begin{bmatrix} K_1 & & \\ & \ddots & \\ & & K_N \end{bmatrix}, \quad f = \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}. \tag{2}$$

The original FETI method assumes that Dirichlet boundary conditions are inherited from the original problem, which is shown in Fig. 3. This fact implies that defects of the stiffness matrices, K_i, may vary from zero, for the sub-domains with enough Dirichlet conditions, to the maximum (6 for 3D solid mechanics problems and 3 for 2D ones) in the case of the sub-domains exhibiting some rigid body modes. General solution to such systems requires computation of generalised inverses and bases of the null spaces, or kernels, of the underlying singular matrices. The problem is that the magnitudes of the defects are difficult to evaluate because this computation is extremely disposed to the round off errors [8].

4. The total FETI method

In this section, we briefly review the main ideas the TFETI method stems from. To circumvent the problem of computing bases of the kernels of singular matrices, Dostál came up with a novel solution [5]. His idea was to remove all the prescribed Dirichlet boundary conditions and to enforce them by the Lagrange multipliers denoted as λ^B in Fig. 4. The effect of the procedure on the stiffness matrices of the sub-domains is that their defects are the same and their magnitude is known beforehand. From the computational point of view such approach is advantageous, see [8] for discussion of this topic.

The overall approach resembles the classic one by Farhat et al. [2] and others, e.g. [3]. The Lagrangian associated with the problem governed by Eq. (1) is as reads

$$L(u, \lambda) = \frac{1}{2} u^\top K u - f^\top u + \lambda^\top B u. \tag{3}$$

This is equivalent to the saddle point problem

$$\text{Find} \quad (\bar{u}, \bar{\lambda}) \quad \text{so that} \quad L(\bar{u}, \bar{\lambda}) = \sup_{\lambda} \inf_{u} L(u, \lambda) \tag{4}$$