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A preconditioning strategy for microwave susceptibility in ferromagnets

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Abstract

3D numerical simulations of ferromagnetic materials can be compared with experimental results via microwave susceptibility. In this paper, an optimised computation of this microwave susceptibility for large meshes is proposed. The microwave susceptibility is obtained by linearisation of the Landau and Lifchitz equations near equilibrium states and the linear systems to be solved are very ill-conditioned. Solutions are computed using the conjugate gradient method for the normal equation (CGN Method). An efficient preconditioner is developed consisting of a projection and an approximation of an "exact" preconditioner in the set of circulant matrices. Control of the condition number due to the preconditioning and evolution of the singular value decomposition are shown in the results.

Keywords: Preconditioning of large systems; Circulant matrices; Micromagnetism

1. Introduction

Ferromagnetic simulation via the micromagnetic model is a real-life computational challenge. Ferromagnetic materials are used in numerous applications such as radar protection, magnetic recording or microelectronics. In these applications, the magnetic objects studied are micro or nano-objects which are difficult and expensive to craft. Thus, one of the optimisation solutions, for the shape and composition of such particles, is numeric simulation. The first step in this type of simulation is to compute the dynamic of the magnetisation and the equilibrium states. However, a direct comparison of the results with experiments is impossible for 3D particles. The main comparison tool is microwave susceptibility as the resonance numerical curves can be compared with the physical experiments. At that point several difficulties are encountered. The main one is managing a large number of degrees of freedom. This is required to compute interesting configurations with sufficient accuracy.

In this article, we use the micromagnetism model in order to model the magnetisation behaviour in ferromagnetic materials. This model is a mesoscopic model, i.e. a model valid for a scale between the one used for microscopic Maxwell equations and the scale of classic macroscopic Maxwell equations. In this model, magnetisation does not linearly depend on magnetic excitation but is controlled by a nonlinear system: the Landau–Lischitz equation (1). This model was introduced by Brown [1,2].

There are two ways to obtain the equilibrium states. The first by energy minimisation ([3–5]), the second by relaxation of the dynamic system [6,7]. The main advantage of the dynamical approach is to compute an equilibrium state linked to given initial data by a life-like dynamic process; then, we can apply dynamical treatments, via the external field, in order to find specific equilibrium states.

Computation of the microwave susceptibility can be performed by two main methods: the harmonic direct computation and the Fourier transform method. The first method is based upon the use of a linearised version of the evolution equation perturbated by a time harmonic external field. The second is based upon the injection of an harmonic perturbation. The Fourier method implies the resolution of a time dependant problem that is quite ill-conditioned for low frequencies (the time step ensuring that the convergence vanishes swiftly when the frequency decreases) but the linearisation methods permit the range of frequencies used in the applications to be attained.

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2. The microwave susceptibility problem

2.1. The linearisation

In this problem, we are interested in computing the microwave response of a ferromagnetic system to an external harmonic excitation. We consider that the ferromagnetic material is homogeneous and contained in a $\mathscr{C}^1$-class piecewise domain of $\mathbb{R}^3$ denoted Ω. Then, we study the evolution of the magnetisation field in the neighbourhood of an equilibrium state of the dynamic equation. This equation, in the micromagnetism model [1], is given by the Landau–Lifchitz system: find m in $\widetilde{H}^1([0,T]\times\mathbb{R}^3,\Omega;\mathbb{R}^3) = \{m\in L^2(\mathbb{R}^3;\mathbb{R}^3)\mid \forall t\in[0,T], m_{|\Omega}\in H^1(\Omega;\mathbb{R}^3)$ and $m\equiv 0$ in $\mathbb{R}\setminus\Omega\}$ such that

$$\begin{cases} \frac{\partial m}{\partial t} = f(m,h_{\text{ext}}) = -m\wedge(H(m)+\ell) \\ \qquad -\alpha m\wedge(m\wedge(H(m)+\ell)), \\ \quad \in (0,T]\times\Omega, \\ m(x,0) = m_0(x) \quad \forall t\in\Omega. \end{cases} \tag{1}$$

where H is a linear operator, from $\widetilde{H}^1([0,T]\times\mathbb{R}^3,\Omega;\mathbb{R}^3)$ into $H^{-1}(\mathbb{R}^3;\mathbb{R}^3)$, ℓ the external magnetic field independent of the magnetisation and element of $L^\infty([0,T]\times\mathbb{R}^3;\mathbb{R}^3)$, α the damping factor (a strictly positive real) and m_0 is a given element of $\widetilde{S}^2(\Omega) = \{m\in\widetilde{H}^1(\mathbb{R}^3,\Omega;\mathbb{R}^3)\| |m_{|\Omega}| = 1$, a.e. in $\Omega\}$. In this model, we can see that the local module of the magnetisation is naturally preserved. In this article, we define H as follows: $\forall m\in\widetilde{H}^1([0,T]\times\mathbb{R}^3,\Omega;\mathbb{R}^3)$

$$H(m) = A\triangle m + H_{\text{d}}(m) + K(m-(m\cdot u)u)$$

where A and K positive real constants and u is an element of $\widetilde{H}^1([0,T]\times\mathbb{R}^3,\Omega;S^2)$ (S^2 designates the unit sphere). The operator H_{d} is defined in the sense of distributions on $\mathbb{R}^3$ by

$$\begin{cases} \text{rot}(H_{\text{d}}(m)) = 0, \\ \text{div}(H_{\text{d}}(m)) = -\text{div}(m). \end{cases}$$

Now, let us define the equilibrium states of system (1).

Definition 1. For a given ℓ in $L^\infty(\mathbb{R}^3;\mathbb{R}^3)$ (independent of time), a magnetisation state m_ℓ, in $\widetilde{H}^1(\mathbb{R}^3,\Omega;\mathbb{R}^3)$ is an equilibrium state if, and only if,

$$f(m_\ell,\ell) = 0, \quad \text{a.e. in } \Omega.$$

Then, for a given equilibrium state m_ℓ, associated to an external state ℓ, we define the microwave susceptibility.

Definition 2. For a given equilibrium state m_ℓ, associated to an external field ℓ, we denote a susceptibility tensor of the order 3 complex matrices $\chi(\ell)$ defined by

$$(\chi(\ell))_{l,k} = -\frac{1}{2T}(\lambda_k, m_l)_{0,\Omega} \quad \forall(l,k)\in\{1,2,3\}^2,$$

with $\lambda_k = \zeta_k e^{i\omega t}$ and ζ_k is a constant vector of $\mathbb{R}^3$. Furthermore, we suppose that $(\zeta_k)_{k\in\{1,2,3\}}$ is an orthogonal basis of $\mathbb{R}^3$. Then, for all k in $\{1,2,3\}$, m_k is a solution of (1) for the external field $\lambda_k+\ell$ and the initial data $m_0 = m_\ell$.

Formally, if the excitation ζ_k is sufficiently small, then the magnetisation responses will be also small and we can define this response for every k in $\{1,2,3\}$ by

$$m_k - m_\ell = \mu_k e^{i\omega t},$$

with $\mu_k\in\widetilde{H}^1(\mathbb{R}^3,\Omega;\mathbb{C}^3)$. In the following we suppose that ζ_k and μ_k are of the same order.

Then, if we re-write system (1) verified by m_k, the linearised equation gives

$$(i\omega - D_{1,\ell}\circ h - D_{2,\ell})(\mu_k) = D_{1,\ell}(\zeta_k) \tag{2}$$

where, for all w in $L^\infty(\mathbb{R}^3;\mathbb{R}^3)$, we set

$$D_{1,\ell}(w) = -m_\ell\wedge w - \alpha m_\ell\wedge(m_\ell\wedge w),$$
$$D_{2,\ell}(w) = (H(m_\ell)+\ell)\wedge w + \alpha m_\ell\wedge(w\wedge(H(m_\ell)+\ell))$$

2.2. The discretisation of the linearised equation

In order to discretise the equation, we consider a monolith $K(\Omega)$ such that $\Omega\subset K(\Omega)$. Ideally, this monolith is the smaller containing Ω. Then, $K(\Omega)$ is discretised using a regular cubic mesh of cells $(\Omega_i)_{i\in N_h}$ where h is the length of a cell and N_h is the set of the indices. We set $\Omega_h = \bigcup_{i\in N_{\text{int},h}}\Omega_i$ where $N_{\text{int},h}\subset N_h$ is the set of indices such that, for every i in $N_{\text{int},h}$, $\Omega_i\cap\Omega\neq\emptyset$.

Then, we choose as a discrete space for all euclidian space F:

$$W_h(F) = \{u\in L^2(\mathbb{R}^3;F)|u\equiv 0 \text{ in } \mathbb{R}^3\setminus K(\Omega) \text{ and}$$
$$\forall i\in N_h,\ u_{|\Omega_i} \text{ is a constant}\},$$

for each u in W_h, we set: $\forall i\in N_h$, $u_i = u_{|\Omega_i}$. We choose the L^2 scalar product on $\mathbb{R}^3$ as the scalar product on W_h, we denote it $(u,v)_{0,\Omega}$ for all u, v in $L^2(\mathbb{R}^3;F)$. Then, setting

$$L^2(\mathbb{R}^3;F)\xrightarrow{P_h} W_h(F)$$
$$u\mapsto P_h(u) = \sum_{i\in N_h}\left(\frac{\mathbf{1}_i}{h^3}\int_{\Omega_i} u\,\mathrm{d}x\right)$$

where $\mathbf{1}_i$ is defined for x in $\mathbb{R}^3$ by $\mathbf{1}_i(x) = 1$ if x belongs to Ω_i, $\mathbf{1}_i(x) = 0$ otherwise. $P_h^\star$ designates the canonical injection of $W_h(F)$ onto $L^2(\mathbb{R}^3;F)$.

These definitions lead to the following formulas for the discrete magnetic contributions:

$$H_{\text{a},h} = P_h\circ H_{\text{a}}\circ P_h^\star,$$

and

$$H_{\text{d},h} = P_h\circ H_{\text{d}}\circ P_h^\star,$$

the analysis of $H_{\text{a},h}$ is straightforward. On the other hand, the analysis of $H_{\text{d},h}$ is not direct, in particular, it has been demonstrated that this discretisation preserves the main properties of the demagnetisation operator H_{d} (H_{d} is a projection operator), and a lower estimate of its lower eigenvalue is given. Furthermore, the computation of this operator is very expensive: the discrete matrix is a full matrix. Then, to optimise its computation, we choose to use a