Environmental Modelling & Software 24 (2009) 449–462

Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

Reliable water supply system design under uncertainty

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ARTICLE INFO

Article history:
Received 25 March 2008
Received in revised form 1 August 2008
Accepted 12 August 2008
Available online 26 November 2008

Keywords:
Water supply system
Data uncertainty
Spatially correlated data
Robust optimization

ABSTRACT

Given the natural variability and uncertainties in long-term predictions, reliability is a critical design factor for water supply systems. However, the large scale of the problem and the correlated nature of the involved uncertainties result in models that are often intractable. In this paper, we consider a municipal water supply system over a 15-year planning period with initial infrastructure and possibility of construction and expansion during the first and sixth year on the planning horizon. Correlated uncertainties in water demand and supply are applied on the form of the robust optimization approach of Bertsimas and Sim to design a reliable water supply system. Robust optimization aims to find a solution that remains feasible under data uncertainty. Such a system can be too conservative and costly. In the Bertsimas and Sim approach, it is possible to vary the degree of conservatism to allow for a decision maker to understand the tradeoff between system reliability and economic feasibility/cost. The degree of conservatism is incorporated in the probability bound for constraint violation. As a result, the total cost increases as the degree of conservatism (and reliability) is increased. In the water supply system application, a tradeoff exists between the level of conservatism and imported water purchase. It was found that the robust optimization approach addresses parameter uncertainty without excessively affecting the system. While we applied our methodology to hypothetical conditions, extensions to real-world systems with similar structure are straightforward. Therefore, our study shows that this approach is a useful tool in water supply system design that prevents system failure at a certain level of risk.

1. Introduction

A water supply system typically includes multiple sources and demand centers (agricultural, domestic, industrial and commercial users). System components are designed to treat relatively good quality source waters from an aquifer or various surface supplies and deliver it to users in a water distribution system that is sized to provide fire flows. Water system design represents a tradeoff between treatment plant size (economy of scale) and the number of plants, pipe and pump sizes and energy consumption, and component sizes, travel time and water quality. To accommodate growth, affected communities have found it necessary to shift from reliance on traditional water supplies—ground water or relatively meager surface flows—toward a combination of large, engineered water projects, water reuse and conservation measures. All of these decisions are made subject to uncertainties introduced by future growth rates and locations, water resource availability, and changing social and institutional conditions.

Much research has been conducted on the simulation of the water supply system (Cai, 2008; Chung et al., 2008; Makropoulos et al., 2008; Mitchell et al., 2008). A municipal water supply system is defined as the physical infrastructure to treat, deliver water to and collect water from users. The design of the capacities of alternative components in a water supply system is usually based upon predictions of future population and climatic conditions. Uncertainty in predicting these conditions is inherent in all water supply systems. Thus, a decision made with a deterministic model that is based on satisfying demand/supply conditions without consideration of uncertainty may result in two consequences: (i) lower net benefits than expected (i.e., it is more costly to provide the desired water) or, (ii) some probability of system failure, where failure is defined as not meeting a given demand or other system constraint (Watkins and McKinney, 1997). These consequences may be rectified in real-time operations at some cost but flexibility must be built into the system during the design process to allow for those adjustments. Deterministic optimization removes this flexibility, thus, a reliability-based design tool is needed that can assist decision makers plan a long-term water supply scheme to cope with the future changes in water demands and supplies.

The complexity of a water system and the correlated uncertainties make incorporating uncertainty a challenging exercise.

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doi:10.1016/j.envsoft.2008.08.007

Nomenclature

Indices and sets

N	set of nodes in the water supply network (sources, users, and treatment plants)
A	set of arcs (i, j) from node i to node j in the network
T	set of design periods, $t \in \mathbf{T}$
O	set of operation periods, $o \in \mathbf{O}$

Data

f	Darcy–Weisbach coefficient
n_{ij}	Manning coefficient of canals from node i to node j
z_{ij}	channel side slope from node i to node j
I	interest rate
CITY	city multiplier
ENR^o	construction cost index at year o
Δo^o	length of operation period o
A_b	basin area
A'_b	basin area contributing to imported water
$A_\mathrm{DO\_OUT}$	outdoor land areas in domestic area
A_AG	land area in agricultural irrigation
L_{ij}	length of arc (i, j)
RQ_i	required discharge at node i in operation year o
WS_i^o	water storage at node i in operation year o
RS_i	required storage at node i in operation year o
EL_i	elevation at node i
$H_{\mathrm{min},ij}$	minimum pressure requirement at the end of pipe and pump connection (i, j)
C_IW	unit cost of purchasing imported water
ρ_i	correlation coefficients from precipitation to water demand and imported water availability
$\varDelta_{ij}^o$	elevation differences at arc (i, j) $(= EL_j - EL_i)$ in operation year o
S_{ij}	channel bottom slope for arc $(i,j) = \varDelta_{ij}/L_{ij}$

Stochastic data

$\tilde{P}^o$	precipitation at operation year o
$\widetilde{\mathrm{IW}}^o$	imported water available at period o
$\tilde{D}_i^o$	demand at node i in operation year o

Decision variables

q_{ij}^o	operation flow rate $[\mathrm{L}^3/\mathrm{T}]$ for arc (i, j) at operation year o
κ_{ij}^t	pipe diameter [L] for arc (i, j) at design period t
d_{ij}^t	canal depth [L] for arc (i, j) at design period t
χ_{ij}^t	pump design capacity for arc (i, j) $[\mathrm{L}^3/\mathrm{T}]$ at design period t
H_{ij}^t	pump design head for arc (i, j) [L] at design period t
w_i^t	capacity of treatment plant $[\mathrm{L}^3]$ at node i at design period t
x_{ij}^t	takes value 1 if a pipe in arcs (i, j) is built at design period t, 0 otherwise
μ_{ij}^t	takes value 1 if a pump in arcs (i, j) is built at design period t, 0 otherwise

A number of stochastic optimization approaches have been applied to water supply system design and operation. Most works have adopted two-stage or multi-stage linear or nonlinear stochastic programming with recourse. The main objectives of these studies were to minimize expected total cost for water transfer to spot-markets (Lund and Israel, 1995); to develop long- as well as short-term water supply management strategies (Wilchfort and Lund, 1997); to manage water supply capacity under water shortage conditions (Jenkins and Lund, 2000); and to design and operate a water supply system (Elshorbagy et al., 1997). On the other hand, a scenario analysis approach for water system planning and management under uncertainty is presented by Pallottino et al. (2005). Recently, two-stage and multi-stage stochastic program technique with interval parameters was applied in water-trading and water resources management by Luo et al. (2007) and Li et al. (2007), respectively. Water-trading problem solved by two-stage stochastic program was applied in the Swift Current Creek watershed in Canada by Luo et al. (2007) and Li et al. (2007) applied inexact multi-stage stochastic integer program in a case study.

Some water supply optimization studies have considered the aspect of system failure risk. For example, Fiering and Matalas (1990) investigated the robustness of water supply planning with respect to global climate change for regions where water storage capacity is limited. Watkins and McKinney (1997) considered uncertainty factors by introducing the standard deviation of the objective function as a constraint into a two-stage stochastic model by Lund and Israel (1995). This is embedded in the robust optimization framework of Mulvey et al. (1995).

Chance-constrained models may explicitly limit the probability of not being able to meet a constraint. Chance-constrained models, while intuitively easy to model, are usually non-convex causing difficulties in optimization and the approach may require numerical integration of the probability distribution. El-Gamel and Harrel (2003) applied chance-constrained genetic algorithm on water supply and irrigation canal systems management.

In this paper, the robust optimization framework of Bertsimas and Sim (2004) is used to develop a reliable water supply system design. A robust solution can be defined as one that remains feasible under uncertainty. This type of robust optimization was first introduced by Soyster (1973) for linear programming problems. Soyster's model significantly constrains the objective function to assure robustness; thus conservative solutions that are found that may be practically unrealistic. Ben-Tal and Nemirovski (1999, 2000), El-Ghaoui and Lebret (1997), and El-Ghaoui et al. (1998) extended the Soyster model. These extensions, however, introduced a higher degree of non-linearity. Since real systems themselves are likely to be nonlinear, these approaches make the problem more complicated and difficult to find a solution. The approach of Bertsimas and Sim controls the degree of conservatism for the system reliability without increasing the difficulty in solving the original problem.

In Section 2, we provide background on the robust optimization framework, followed by the water supply model formulation with uncertain data. Next, we present the details of our uncertainty model and the robust formulation. Tradeoff between system reliability and economic feasibility using the degree of conservatism is demonstrated in a municipal water supply system application. Finally, we end with conclusions and future research.

2. Robust optimization framework

The classical assumption in deterministic mathematical programming is that all parameters (input data) are known precisely. This is rarely the case in real applications since many parameters contain uncertainties such as in future predictions or in measurement. One way to deal with uncertainty is to design a system that is "robust" to parameter changes. That is, the system remains feasible and operates in a near-optimal fashion for a variety of values that the uncertain parameters can take. Soyster (1973) formulated the following linear programming model to find