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Nonlinear Measurement Update for Recursive Filtering Based on the Gauss von Mises Distribution

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Abstract

In conventional Kalman-based state estimation algorithms, there is an assumption that the uncertainties in the system state and measurements are Gaussian distributed. However, this Gaussian assumption ignores the periodic nature of angular or orientation quantities. In this paper, the Gauss von Mises (GVM) distribution model defined on a cylindrical manifold is employed, the Dirac mixture approximation method is extended to deal with sampling with GVM, in order to perform recursive filtering, the GVM approximation to joint distribution is proposed, the formula to compute posterior distribution is derived.Finally, the measurement update algorithm is developed. Simulation results show that when the system state contains a circular variable, the proposed GVM filter can achieve more accurate estimates than the traditional extended Kalmanfilter(EKF), thereby providing a novel method to estimate system state specialized to GVM distribution.

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1. Introduction

Accurate estimation of system state is crucial in many applications such as navigation, motion estimation. Since most of the filters including standard Kalman-based filters adopt Gaussian assumptions, the results are not satisfactory when the state vector contains periodic angular or orientation variables, especially when the measurements from low-cost sensorsinvolves significant uncertainties. Therefore, we need more accurate models and filtering methods to take into account the underlying structure of the state space.

Various probability distribution models defined on different manifolds are available in the literature. The generalization of von Mises distribution to cylindrical manifold[1], multivariate von Mises distribution[2], partially wrapped normal(PWN) distribution for rigid motion group SE(2)[3]are proposed, but corresponding state estimation algorithms are not given in these work. Circular fusion filter[4], quaternion Bingham filter[5] are also proposed for the circular filtering problem.However, these filters only consider the case when the state isa single circular variable or 3D rotation represented by a quaternion.Gauss von Mises(GVM) distribution is proposed by J.T.Horwood[6,7]to deal with the space surveillance tracking environment that is typically nonlinear and non-Gaussian. Itcan characterize the uncertainty in the orbital state of a space object. But in their work, only uncertainty propagation problem has been considered, i.e., the time update step in Bayesian filtering, without presenting algorithms for the measurement update step.

The rest of the paper is organized as follows. The GVM distribution is introduced in Section 2.The Dirac mixture approximation method is extended to enable sampling with GVM distribution in Section 3. In Section 4, the GVM approximation to joint distribution is proposed, the formula for calculating GVM parameters for posterior distribution is derived, and the measurement update algorithm is developed. In Section 5, simulation results are presented. Finally, conclusions are drawn in Section 6.

2. Definition of the GVM Distribution Model

Definition 1. (Gauss von Mises (GVM) Distribution)[6]. The random variables $(\theta, \boldsymbol{x}) \in \mathbf{S} \times \mathbf{R}^n$ are said to be jointly distributed as a Gauss von Mises (GVM) distribution if and only if their joint PDF has the form:

$$p(\boldsymbol{x},\theta) = GVM(\boldsymbol{x},\theta;\boldsymbol{\mu},\boldsymbol{P},\alpha,\boldsymbol{\beta},\boldsymbol{\Gamma},\kappa) \equiv N(\boldsymbol{x};\boldsymbol{\mu},\boldsymbol{P})VM(\theta;\Theta(\boldsymbol{x}),\kappa) \tag{1}$$

where:

$$VM(\theta;\Theta(\boldsymbol{x}),\kappa) = \exp(\kappa\cos(\theta-\Theta(\boldsymbol{x})))/2\pi I_0(\kappa) \tag{2}$$

$$\Theta(\boldsymbol{x}) = \alpha + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{y} + (1/2)\boldsymbol{y}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{y},\ \ \boldsymbol{y} = \boldsymbol{L}^{-1}(\boldsymbol{x}-\boldsymbol{\mu}),\ \ \boldsymbol{P} = \boldsymbol{L}\boldsymbol{L}^{\mathrm{T}}. \tag{3}$$

The parameter set $(\boldsymbol{\mu},\boldsymbol{P},\alpha,\boldsymbol{\beta},\boldsymbol{\Gamma},\kappa)$ should meet the following constraints: $\boldsymbol{\mu} \in \mathbf{R}^n$, $\boldsymbol{P}$ is an $n \times n$ symmetric positive-definite matrix, $\alpha \in \mathbf{R}$, $\beta \in \mathbf{R}^n$, $\boldsymbol{\Gamma}$ is an $n \times n$ symmetric matrix, and $\kappa > 0$. The matrix $\boldsymbol{L}$ is the lower-triangular Cholesky factor of $\boldsymbol{P}$.

The notation $(\boldsymbol{x},\theta)$: $GVM(\boldsymbol{\mu},\boldsymbol{P},\alpha,\boldsymbol{\beta},\boldsymbol{\Gamma},\kappa)$ is used to represent that $(\boldsymbol{x},\theta)$ are jointly distributed as a GVM distribution with parameter set $(\boldsymbol{\mu},\boldsymbol{P},\alpha,\boldsymbol{\beta},\boldsymbol{\Gamma},\kappa)$. We can apply the transformation: $\boldsymbol{y} = \boldsymbol{L}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})$, $\phi = \theta - \alpha - \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{y} - (1/2)\boldsymbol{y}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{y}$ to transform $GVM(\boldsymbol{\mu},\boldsymbol{P},\alpha,\boldsymbol{\beta},\boldsymbol{\Gamma},\kappa)$ to canonical GVM distribution with:

$$p(\boldsymbol{y},\phi) = GVM(\boldsymbol{y},\phi;\mathbf{0},\mathbf{I},0,\mathbf{0},\mathbf{0},\kappa) = N(\boldsymbol{y};\mathbf{0},\boldsymbol{I})VM(\phi;0,\kappa) \tag{4}$$