Journal of Neuroscience Methods 275 (2017) 33–44

Contents lists available at ScienceDirect

Journal of Neuroscience Methods

journal homepage: www.elsevier.com/locate/jneumeth

Total variation for the analysis of event-related potentials

Alexander Klein*, Wolfgang Skrandies

Justus-Liebig-Universität Gießen, Physiologisches Institut, Aulweg 129, 35392 Gießen, Germany

HIGHLIGHTS

- An analytical measure capable of integrating different aspects of signal morphology is introduced.
- We demonstrate its usefulness for the analysis of event-related potential waveforms.
- It can detect effects of experimental manipulation in the absence of obvious peaks.

ARTICLE INFO

Article history:
Received 6 April 2016
Received in revised form 17 October 2016
Accepted 17 October 2016
Available online 26 October 2016

Keywords:
Total variation
Event-related potential
Time-domain
Learning
Feedback
tANOVA

ABSTRACT

Background: Event-related potential waveforms are often analysed in the time-domain for changes of striking morphological features, like amplitudes or latencies of extrema, at the expense of missing less obvious changes in overall morphology.
New method: The measure of *total variation* can capture a variety of changes in curve morphology. We show analytical examples, and the application to two sets of EEG data ($n_1 = 41$, $n_2 = 19$) difficult to analyse with more traditional methods.
Results: Total variation can be used to identify the effects of experimental manipulations on event-related potential waveforms, and can additionally be used to identify the respective time windows by means of hierarchical subdivision of longer signals.
Comparison with existing methods: The ANOVA of total variation provided additional insights into effects already hinted at by the ANOVA of global field power in the first experiment, and identified a number of interactions missed by an ANOVA of amplitude as well as a topographic ANOVA in the second one.
Conclusions: The analysis of total variation can be an interesting complement to more traditional analyses, especially when changes are hard to assess with traditional methods, e.g. in the absence of pronounced extrema, or the presence of noise or large interindividual variations of latency.

1. Introduction

Event-related potential waveforms (ERPs) are commonly analysed in the time-domain for *prima facie* features of the curve, like peak amplitudes and latencies, or their respective differences. In the frequency-domain, on the other hand, it is possible to analyse ERPs as a whole for changes in amplitude or phase at certain frequencies.

Conventional methods in the time-domain are usually completely ignorant with respect to features in the frequency-domain and vice versa. It is usually not possible, for example, to detect changes of frequency content by analysing amplitudes, latencies, or simple moment estimates like mean and variance, whereas in a Fourier-analysis, shifts in peak-latency could only be detected as obscure changes of phase over the whole range of frequencies, which may remain undetected, as the phase-spectrum is often not looked at, leaving only changes of the amplitude-spectrum as a subject of analysis.

Wavelet- and Gabor-analysis or similar methods (Witte et al., 2008; Wacker and Witte, 2013) may provide a way out of this dilemma by merging time- and frequency-domain analysis, but doing statistics correctly may prove challenging in this context (Maraun et al., 2007), and it is not easy to decide what exactly to search for: complex wavelets provide amplitude- and phase-scalograms, but while the information provided by the former is quite obvious, as in their simplest form they can indicate, whether a certain “frequency” is “present” at a given time, the latter are only poorly understood, although useful information can be inferred from them (Deng et al., 2005a,b).

While all these are well-established methods of analysis, many of them tend to focus very narrowly on specific features like peaks and their latencies, perhaps within given regions of interest, while

* Corresponding author.
E-mail address: alexander.klein@physiologie.med.uni-giessen.de (A. Klein).

http://dx.doi.org/10.1016/j.jneumeth.2016.10.012
0165-0270/

ignoring everything else. On the other hand, trying to analyse ERPs as a whole by simply treating them as stochastic processes and estimating their moments is not effective, either, as moment estimates have difficulty representing the overall *morphology* of a signal.

ERPs often show considerable interindividual variation. Aside from obvious differences in mean, variance, extremal amplitudes, and feature latencies, there may also be differences in the general shape of components: temporal dilatation or contraction may occur, additional superimposed fast undulations might undergo changes in amplitude, frequency or phase, or additional local extrema on the shoulders of larger components might occur or change. Such changes may occur alone or simultaneous, and may or may not influence the above measures.

After all, it seems desirable to have an additional measure quantifying overall signal morphology, as opposed to measures for specific features only. We think that one useful descriptor for such a task is a measure called "total variation". In the following, we will give an analytical definition of this measure, work out a few analytical examples, show a few simulations comparing its performance to other measures, and finally apply it for the analysis of data taken from two EEG experiments. The results show that the analysis of total variation may provide additional insight into morphological changes of signals that might otherwise remain unnoticed, thus complementing traditional methods of analysis with little additional effort in computation.

Although the concept of total variation is by no means new, it seems to have largely gone unnoticed as a general means for signal analysis. The concept itself appears under the name "short time line length" in Dümpelmann et al. (2012, p. 1723) as a means for detection of high frequency ripples in epilepsy, and a similar concept has been used as an intermediate step for fractal analyses of waveforms (Katz, 1988). However, we currently know of no application of the measure itself for the evaluation and comparison of signal morphologies.

2. Analytical total variation

2.1. Definition and elementary properties

Let $[a, b] \subset \mathbb{R}$ be an interval and $f : [a, b] \to \mathbb{R}$ any function, then the total variation of f is defined as (Sohrab, 2003, p. 284, Def. 7.6.1)

$$\sup \left\{ \sum_{k=1}^{n} \left| f(x_k) - f(x_{k-1}) \right| : n \in \mathbb{N}, a = x_0 < \ldots < x_n = b \right\}.$$

For any continuously differentiable f on an interval $[a, b] \subset \mathbb{R}$, the total variation can be calculated as (Sohrab, 2003, p. 295)

$$V_{[a,b]}(f) := \int_a^b \left| f'(x) \right| \, dx. \tag{1}$$

These somewhat abstract formulæ can be explained immediately, when we restrict ourselves to continuously differentiable functions with only a handful of extrema ("smooth functions") for the sake of simplicity:

Suppose we have a function $f : [a, b] \mapsto \mathbb{R}$ with n extrema at $x_1, \ldots, x_n \in (a, b) \in \mathbb{R}$—which are the points where f' changes its sign, and thus maxima or minima occur—and set additionally $x_0 = a$, $x_{n+1} = b$. In this case we can write

$$\int_a^b \left| f'(x) \right| \, dx = \sum_{k=1}^{n+1} \left| f(x_k) - f(x_{k-1}) \right|, \tag{2}$$

which is just the sum of the differences of the ordinates at adjacent extremal points including the borders, or in other words, the total distance an imaginary pen drawing the curve would travel in y-direction. As an immediate consequence, we see that a second function $g : [\bar{a}, \bar{b}] \mapsto \mathbb{R}$ with n extrema at $\bar{x}_1, \ldots, \bar{x}_n \in (\bar{a}, \bar{b})$ and $f(x_k) = g(\bar{x}_k)$ for all k will have the same total variation, although the intervals $[a, b]$ and $[\bar{a}, \bar{b}]$ may differ in length.

In the case of digitised data $D = (d_0, \ldots, d_n)$, we do not need to determine any extrema at all, as we can approximate Formula (1) directly: Setting Δ as the operator computing the first order differences $d_k - d_{k-1}$, and remembering that the 1-Norm is defined as $\| (y_1, \ldots, y_n) \|_1 := \sum_{k=1}^{n} |y_k|$, we get the following approximation of Formula (1)[1]:

$$V_{[a,b]}f \approx \| \Delta D \|_1,$$

because Δ approximates the first derivative, and $\|\cdot\|_1$ approximates the integral of the absolute value, both with reciprocal factors of proportionality. For a given vector of recorded data, we can therefore estimate $V_{[a,b]}f$ directly by means of the simple command `norm(diff(D),1)` in GNU-Octave (Octave community, 2012), for example.

2.2. Examples

2.2.1. Analytical examples

A polynomial. Let $f(x) := (x-1)\cdot(x-2)\cdot(x-3) = x^3 - 6x^2 + 11x - 6$. What is the total variation of f on $[0, 4]$?

After differentiating we get $f'(x) = 3x^2 - 12x + 11$, and the extremal abscissæ can be determined as the zeroes thereof, which are $x_{1,2} = 2 \pm 1/\sqrt{3}$. Using Formula (2) we get

$$V_{[0,4]}f = \sum_{k=1}^{3} |f(x_k) - f(x_{k-1})| = \left| \frac{2}{3\sqrt{3}} - (-6) \right| + \left| -\frac{2}{3\sqrt{3}} - \frac{2}{3\sqrt{3}} \right| + \left| 6 - \left(-\frac{2}{3\sqrt{3}} \right) \right| = 12 + \frac{8}{3\sqrt{3}} \approx 13.54.$$

Harmonic oscillations. Let $g(x) := \sin(k \cdot x)$, $k \in \mathbb{N}$. What is the total variation of g on an interval $[a, b] \subset \mathbb{R}$?

We simplify the calculations somewhat by first noting that the total variation of a complete cycle of a (co)sine is 4, and that a complete cycle of g happens on an interval of length $2\pi/k$. Now let $x_a := \lceil a/(2\pi/k) \rceil \cdot 2\pi/k \geq a$ and $x_b := \lfloor b/(2\pi/k) \rfloor \cdot 2\pi/k \leq b$ the first start and last end, respectively, of such a complete cycle in the vicinity of $[a, b]$, and $n := (x_b - x_a)/(2\pi/k) \in \mathbb{Z}, n \geq -1$. Basically, n counts the number of complete cycles in $[x_a, x_b]$, but since we never required $x_a < x_b$, n may take the values -1 or 0 as well. Putting the pieces together, we get

$$V_{[a,b]}g = \int_a^{x_a} k \left| \cos kx \right| \, dx + 4n + \int_{x_b}^{b} k \left| \cos kx \right| \, dx.$$

In order to work out an example, take $a := 0.5$, $b := 17.4$ and $k := 7$. In this case we have $x_a = 1 \cdot 2\pi/7$, $x_b = 19 \cdot 2\pi/7$ and $n = 18$. The final result is thus

$$V_{[0.5,17.4]} \sin 7x = \int_{0.5}^{2\pi/7} 7 \left| \cos 7x \right| \, dx + 4 \cdot 18 + \int_{19 \cdot 2\pi/7}^{17.4} 7 \left| \cos 7x \right| \, dx \approx 1.65 + 72 + 1.34 = 74.99.$$

Note that the concept described above is *not* equivalent to the concept of "curve length" used by Katz (1988). In order to compute "curve length", roughly representing the visual length of a

[1] Note, however, that the quality of approximation is dependent on the smoothness of the data, which is directly related to the sampling frequency: The formulæ work well when the maximal frequency in the data is well below the Nyquist limit, as is the case for the data analysed in this paper and probably most real recordings of EEG or ERP signals, but results will get worse as high frequencies and noise creep up, in which case more elaborate approximations would have to be used.