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On ecosystems dynamics

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ABSTRACT

We show how a dynamical system given by a t-score function for some class of monotonic data transformations generates consistent extreme value estimators. The variation of their values increases the uncertainty of proper assessment of climate change. Two important examples illustrate the methodology: mass balance measurements on Guanaco glacier, Chile, and extreme snow loads in Slovakia. We experience singular learning of the transitions in ecosystems.

1. Introduction

In the past several decades scientific effort has been focused on studying and understanding global climate changes. The effect of climatic changes has become more and more visible and in many regions of the world these changes are represented by increasing of weather extremes (Chan et al., 1873; Coumou and Rahmstorf, 2012; Klein Tank and Können, 2003).

All ecosystems (Methan (Sabolova et al., 2015), Guanaco Glacier (Jordanova et al., 2016), Snow extremes (Stehlík et al., 2015)) are oscillating. Decomposition to deterministic, stochastic and chaotic part have been studied by Stehlík et al. (2016). We can understand contributions to oscillations in at least three following ways:

(1) Extreme Value Index (EVI) ξ oscillates around 0 (it can have positive, negative or zero values). As Penalva et al. (2016) pointed out, difficulties may rise with the "Regularity conditions" for the maximum likelihood (ML) estimation (Smith, 1985), it is shown that the usual property of asymptotic normality holds provided the extreme value parameter ξ is larger than -0.5. For all environments we can consider $\xi > -1$ (Penalva et al., 2016). Recently, Zhou (2009, 2010) showed that the ML estimators verify the property of asymptotic normality for $\xi > -1$. The Second Order Regularity condition (SOC) can be difficult to be checked (or even satisfied) in practical application. E.g. if the observed random variable (r.v.) is a power of Uniform or has power law behavior at the finite right end point (see Example 3.3.15 and 3.3.16, page 137, Embrechts et al., 1987), there is not unique SOC parameter ρ.

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(2) Aside of (1), the consistent estimators of tail parameters can be built up upon t-scores (Jordanova et al., 2016). The parameters of harmonic mean estimators (HME) are consisting dynamical system which can surprisingly always find a monotonic representing data function (t-score function) η. This process contributes to deterministic dynamics of Stehlík et al. (2016).
(3) The use of Negative t-Hill(n-t-Hill) for estimation of the EVI index $\xi<0$) can give several limiting behaviors, however, limits can be given by symmetric (normal) or classical (Weibull) distributions, which both are special cases of generalized gamma distribution (ggd), see Stehlík (2008).

The paper is organized as follows. In the next session we study autonomous system of t-score functions. In Section 3 we study mass balance measurements from Guanaco glacier and we show that both negative and positive EVIs are obtained. In Section 4 we study the extremal snow loads in Slovakia, again receiving both negative and positive EVIs. To maintain the readability of the manuscript we put technicalities to Appendix.

2. Dynamical systems of t-score functions

The transformation-based score (Fabián, 2001; Stehlík et al., 2010) or shortly the t-score for the density f is defined as

$$T_\eta(x;\theta) = -\frac{1}{f(x;\theta)}\frac{d}{dx}\left(\frac{1}{\eta'(x)}f(x;\theta)\right).$$

It expresses a relative change of a basic component of the density, i.e., density divided by the Jacobian of mapping η. The t-score is a suitable function for using the generalized moment method for the estimation of parameters of heavy-tailed distributions. Let $X_1,\ldots,X_n$ be independent identically distributed (i.i.d) sample from F with probability density function (p.d.f.) f. The parametric version of the so-called t-mean, which can be considered as a measure of central tendency of distributions, yields the moment estimation equations for θ in the form

$$\frac{1}{n}\sum_{i=1}^{n}T(X_i;\theta)=0.$$

The solution $\hat{\theta}$ is strongly consistent and asymptotically normal (see Fabián, 2001). For t-Hill estimator (Fabián and Stehlík, 2009), we have bounded score

$$S(x;\theta) = T_{\tilde{\eta}}(x;\theta) = \theta\left(1-\frac{\theta+1}{\theta x}\right)$$

and for generalized t-Hill estimator (Beran et al., 2014) (Pareto distribution and $\tilde{\eta}(x)=\ln(x-1), x>1$), we have the score

$$S(x;\theta,\beta)=\begin{cases}\theta\left(1-\dfrac{\theta+\beta-1}{\theta x^{\beta-1}}\right), & \text{for } \beta\neq 1,\\ \theta\left(\dfrac{1}{\theta}-\ln x\right), & \text{for } \beta=1.\end{cases} \tag{1}$$

where $\beta>0$ is tuning parameter. For $\beta=2$ we obtain t-Hill, with "typical" transformation of the support of the distribution $(1,\infty)$ to the whole real line $(-\infty,\infty)$ is $\tilde{\eta}(x)=\ln(x-1), x>1$. Here an important inverse problem arises. For a given score $\tilde{S}$, does there exist one or several sufficiently smooth functions η such that equation

$$T_\eta=\tilde{S} \tag{2}$$

holds? Which qualitative properties do they posses?

Consider now the Pareto distribution with the probability density function (p.d.f.)

$$f(x,\theta)=\theta x^{-\theta-1}, \quad x>1$$

where $\theta>0$ is a shape parameter (the tail index). Let us modify Eq. (2) by multiplying by $f>0$ in order to receive exact 2nd-order differential equation in the form

$$h(x)+\frac{d}{dx}\left(\frac{f(x)}{\eta'(x)}\right)=0, \tag{3}$$

where $h(x)=S(x;\theta,\beta)f(x)$. Now, integrate Eq. (3) to obtain an equation, which is solvable by quadrature, of the form

$$H(x)+\left(\frac{f(x)}{\eta'(x)}\right)=C,$$

where $H(x)$ is an antiderivative of h. Its form (under the condition $\beta\neq 1-\theta$) is:

$$H(x)=\theta^2\int\left(1-\frac{\theta+\beta-1}{\theta x^{\beta-1}}\right)x^{-\theta-1}dx=\theta x^{-\theta}(x^{1-\beta}-1)+C_1.$$

This yields several classes[1] of solutions expressible in general in the form of special functions (a non-elementary antiderivatives). But this is an obstacle, since they can be hardly jointly analyzed because of their transcendental nature.

These difficulties motivate us to study Eq. (2), by a different approach, applicable for general density f and score function $\tilde{S}$. In order to analyze it is more convenient to define some extra variables $\mathbf{w}=(x,y,z):=(t+a,\eta,\eta'), a\in\mathrm{supp}(f)=\{x\in\mathbb{R},: f(x)\neq 0\}$. Under the assumption $\eta'\neq 0$ Eq. (2) is equivalent to the system $\dot{\mathbf{w}}=\mathbf{W}(x,y,z)$, where $\mathbf{W}(x,y,z)=(1,z,\Psi(x,z))$, $\Psi(x,z)=z^2S+z\frac{d}{dx}\ln(f(x))$ and $(x,y,z)\in D_0$, with $\mathcal{D}_0:=[a,\infty)\times[a,\infty)\times\mathbb{R}\backslash\{0\}$.

We use this approach in details for (3), where $a=1, x\geq 1$ is the independent variable, $\eta(x)\geq 1$ is the unknown function with $\eta'(x)\neq 0$ and $(\beta,\theta)\in\mathbb{R}_+$ are parameters. In this way, (3) is equivalent to the following set of autonomous ordinary differential equations:

$$\begin{cases}\dot{x}=1,\\ \dot{y}=z,\\ \dot{z}=\varphi(x,z),\end{cases} \tag{4}$$

where

$$\varphi(x,z)=-\frac{\theta+1}{x}z+\theta\left(1-\frac{\theta+\beta-1}{\theta x^{\beta-1}}\right)z^2,$$

and $(x,y,z)\in\mathcal{D}_0$.

In our setting any initial condition $(x_0,y_0,z_0)\in\mathcal{D}_0$ defines a unique smooth solution of (4)—and, hence, a unique differentiable solution $y=\eta(x)$ of (3). Each solution of (4) can be represented as a smooth orbit $\{(x(t), y(t), z(t))\}$ in $\mathbb{R}^3$ parameterized by $t\in\mathbb{R}$; see Guckenheimer and Holmes (1986) for more details.

The (unique) orbit through a given point $(x,y,z)\in\mathcal{D}_0$ is tangent to the vector $(1, z, \varphi(x, z))$ at the point (x, y, z). Hence, an orbit always flows forward in the direction of x and never "comes back" near any point already visited in the same orbit. More precisely, there is no dense orbit of (4) in any open region of the phase space $\mathbb{R}^3$. Hence, there cannot be topological mixing, which is one of the necessary ingredients of chaotic dynamics (Guckenheimer and Holmes, 1986; Hasselblatt and Katok, 2003).

For the fixed initial condition, we are able to obtain a monotonic solution for t-score for almost all possible cases of parameters. The t-score defines consistent estimator of tail parameter θ. The choice of parameter β is an issue of experience for the statistician/

[1] E.g. for $\beta=1$ (Hill or MLE estimator) $\tilde{\eta}(x)=-\theta\ln\ln x+\text{const.}, x>1$ is the example of η which can be expressed in terms of elementary functions.