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Reconstructing minimal length tree branch systems from leaf positions

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ABSTRACT

We present a method to infer a straight-lines tree branch system from a given set of leaf positions and average branching angles. Among an extensive set of possible branch systems constructed in the process, we choose the one featuring the shortest total length, following an optimality hypothesis by Leopold (1971). The approach is illustrated using empirical low-order skeletons from European beech. Our method further allows to assess, for a given species or individual tree, to what extent its branching pattern accords to Leopold's hypothesis, which we argue to be the case for beech. While yet facing issues of computational intensity for too many leaves, the method can furthermore be used to complement existing tree structure reconstruction methods that otherwise require a rudimentary skeleton as manual input.

1. Introduction

The reconstruction of tree structures based on information from photographs (Shlyakhter et al., 2001; Tan et al., 2008) and point clouds obtained via LiDAR (Zhu et al., 2008; Yan et al., 2009; Livny et al., 2010; Preuksakarn et al., 2010; Raumonen et al., 2013) for computer graphics or structural plant modelling has been an active research area in recent years. The underlying data often merely captures tree's foliage enveloping the crown, which requires additional processing to reconstruct the inner-crown branch system. Some of these methods require a rudimentary, low branching order tree skeleton as user input: in the model by Neubert et al. (2007), a branch structure is formed by the paths of points starting at the leaf positions and being transported down to the stem base along a vector field that is defined based on an a priori given trunk and primary branch structure. On the same basis, Sakaguchi (1998) used L-systems to generate a more detailed branch skeleton that fills a given crown envelope; Tan et al. (2008) proceeded similarly. Our objective in this paper is to develop a method that allows to construct a skeleton from scratch and can thus be used to complement the above-mentioned approaches. This ties in with similar recent methods that have used a space colonisation algorithm (Runions et al., 2007) and particle systems (Rodkaew et al., 2003; Owens et al., 2016) to reconstruct realistic plant structures.

In his seminal article for the systematisation of tree structure, Leopold (1971) noted that “by analogy [to river systems] it seems possible that the branching patterns of trees and of other biologic forms are governed by […] tendencies which are analogous to minimum energy expenditure […]. In the case of trees it might be supposed that [this] involves minimising the total length of all branches and stems”, and hypothesised that “the form which is most probable also tends to minimise the total length of all paths within the applicable constraints”. Hence, for a given distribution of leaves, he assumes the minimisation of the total length of the branch system and thus of wood mass, implying a maximisation of leaf mass and thus future biomass production. Leopold's point of view is teleonomic, in that it supposes the lower hierarchical processes of branch longitudinal growth and ramification to be governed by a higher hierarchical goal (Thornley and Johnson, 2000). This general approach of an assumed goal-directedness of growth processes, often towards the maximisation of growth variables (Dewar, 2010) is common in many plant models. Fisher (1992) and Farnsworth and Niklas (1995) revisited architectural models in the context of optimising light interception, while Mäkelä et al. (2002) provided a review of functional teleonomic and optimisation models at the plant level. Canell and Dewar (1994), Le Roux et al. (2001) and McMurtrie and Dewar (2013) addressed teleonomy in the context of carbon allocation models.

Here, we present a method for the reconstruction of a branch system in terms of line segments based on a given set of leaf positions as well as stem base, which may have been obtained from photographs or LiDAR data, in a manner that is strongly based on Leopold's hypothesis. We

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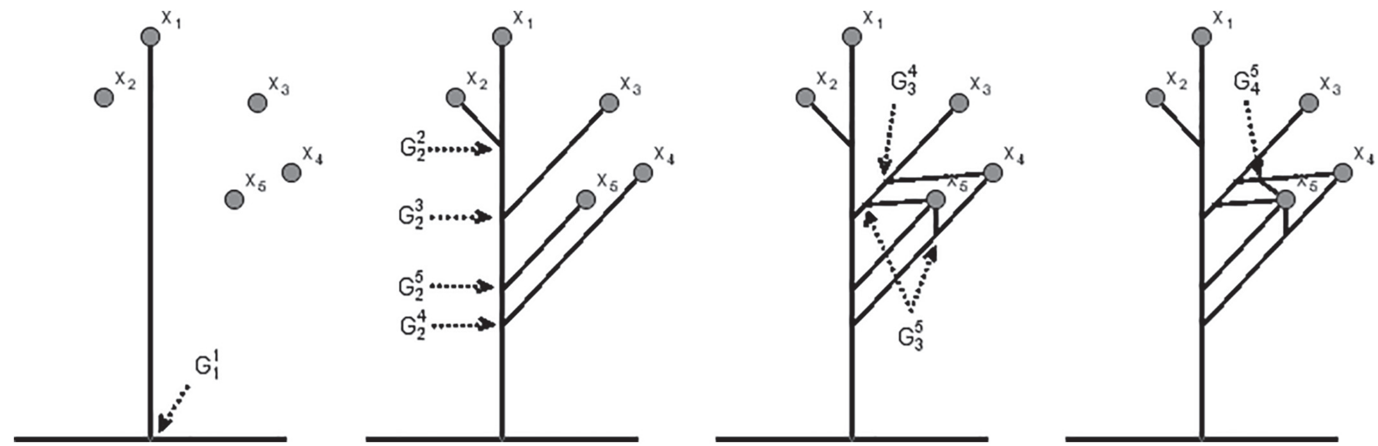

Fig. 1. In step $n > 1$, leaves are connected to branch structures established in the previous step in accordance with the criteria (2) and (3). Here we used $\varphi_n = 137.5°$ for all n. Below the figure: Topologies corresponding to the branching points G_n^i. For instance, in step 3, a branch connection from $\mathbf{x_4}$ is drawn to a branch leading to $\mathbf{x_3}$ which, on its part, had been connected to a branch to $\mathbf{x_1}$ in the previous step. Hence $431 \in T_3^4$. For $n \geq 5$, $T_n^i = \varnothing$ for all i.

first construct a set of possible branch systems that connect the leaf positions to the stem base via line segments and bifurcations, and which is essentially formed only under the constraint of given branching angles. This set covers a large set of skeletons, of which, in the end, we identify the system with the shortest total length.

2. Method description

We follow Honda (1971) in that branch segments are straight. A necessary input for our method are branching angles, i.e. the angle between a mother and daughter branch. We assume these to be only dependent on branching order, which is an assumption common in many functional-structural tree models (Takenaka, 1994; Perttunen et al., 1996; Grote, 2002). It can principally be relaxed, e.g. by introducing a dependence on the ratio of height and crown diameter of the given leaf density as a global proxy for competition. Denote by φ_n the bifurcation of an order n branch from an order $n-1$ branch in the botanical (or Hack's) ordering system (Borchert and Slade, 1981).

The starting point of the following procedure is a given line segment representing the trunk. In the first step we construct line segments leading from the leaf positions to the trunk. In each following step, we then construct new line segments that connect the leaves to branching points on line segments formed in the previous step, at the given branching angle. We thus construct progressively higher-order branches until this is no longer possible. Finally, of all the possible branch systems generated in this manner, we chose the one with the shortest total length.

Next, we describe the method in detail. Although the basic ideas are generally intuitive, the formalism is at times rather cumbersome. Figures along the text illustrate the specific steps for an exemplary set of leaf positions.

2.1. Preliminary definitions

Let $\mathbf{x_1}, \ldots, \mathbf{x_m} \subset \mathbb{R}^2 \times \mathbb{R}_+$ denote the 3D leaf positions of a tree with a trunk rooting, without loss of generality, in (0,0,0). For the sake of convenience we assume $\mathbf{x_1}$ to denote the leaf at the tip of the stem (tree top).

For later use, let $\mathbb{L}^n$ denote the set of ordered lists of length n, composed of natural numbers. Elements of $\mathbb{L}^n$ will later be used to describe the topology of a skeleton reconstruction up to branch order n. For lists $a = a_1a_2 \ldots a_n \in \mathbb{L}^n$ and $b = b_1b_2 \ldots b_m \in \mathbb{L}^m$, with $a_i, b_i, n, m \in \mathbb{N}$, we define the concatenation of a and b by

$$a.\ b = a_1 \ldots a_n b_1 \ldots b_n \in \mathbb{L}^{n+m}.$$

a is by definition contained in b, $a \subset_{\mathbb{L}} b$, if a is a tail of b, i.e. if

$$m \geq n \quad \text{and} \quad a_i = b_{m-n+i} \quad \text{for} i = 1, \ldots, n.$$

a and b are called compatible, $a \sim_{\mathbb{L}} b$, if either $a_i \neq b_j$ for all i,j, or

$$\exists\ i_0 \geq 0: \quad a_{n-i} = b_{m-i} \text{for all} i = 0, \ldots, i_0$$
$$\text{and} \{a_1, \ldots, a_{n-i_0-1}\} \cap \{b_1, \ldots, b_{m-i_0-1}\} = \varnothing.$$

Compatibility is a symmetric and reflexive but not transitive relation. The notion will become intuitive in the following sections.

For two points $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ denote by $\mathbf{x}\ \mathbf{y}$ and $\overline{\mathbf{xy}}$ the line and line segment through $\mathbf{x}$ and $\mathbf{y}$, respectively. With $P_{\mathbf{y,z}}(\mathbf{x}) = \mathbf{z} + (\mathbf{x} - \mathbf{z}) \cdot (\mathbf{y} - \mathbf{z}) \cdot \frac{\mathbf{y} - \mathbf{z}}{\| \mathbf{y} - \mathbf{z} \|^2}$ denoting the orthogonal projection of $\mathbf{x}$ onto the line through $\mathbf{y}$ and $\mathbf{z}$, let

$$P_{\mathbf{y,z}}^{\varphi}(\mathbf{x}) = P_{\mathbf{y,z}}(\mathbf{x}) - \frac{\mathbf{y} - \mathbf{z}}{\|\mathbf{y} - \mathbf{z}\|} \cdot \frac{\|P_{\mathbf{y,z}}(\mathbf{x}) - \mathbf{x}\|}{\tan(\pi - \varphi)}$$

be the oblique projection, where the angle that the line $\mathbf{x}\ P_{\mathbf{y,z}}^{\varphi}(\mathbf{x})$ draws with the line $\mathbf{y}\ \mathbf{z}$ is φ.

2.2. The sets of branching points and topological histories

In the following, we motivate the sets G_n^i and T_n^i, formally defined in Eq. (1) and illustrated in Fig. 1. The set $G_{n+1}^i \subset \mathbb{R}^3$ contains all possible points at which an order $n+1$ branch, that terminates at the leaf in $\mathbf{x_i}$, bifurcates from an order n branch that was constructed in the preceding iteration step. The latter, in turn, is given by a line segment between a leaf in some $\mathbf{x_j}$ and an appropriate branching point on an order $n-1$ branch, etc. Hence, branching points in the set G_{n+1}^i are oblique projections of the leaf in $\mathbf{x_i}$ onto an order n branch that terminates at some leaf in $\mathbf{x_j}$ and starts at some lower-order branching point contained in the set G_n^j. In order for such an oblique projection, given by $P^{\varphi_{n+1}}_{(G_n^j)_k, \mathbf{x_j}}(\mathbf{x_i})$ for some k, to be indeed added to the set G_{n+1}^i, it must satisfy the following two conditions. First, it is required to lie on the line segment (not just the line) between $\mathbf{x_j}$ and $(G_n^j)_k$. Second, the potential new daughter branch must not be longer than the part of the mother branch leading from the potential bifurcation point to the leaf in $\mathbf{x_j}$. The latter condition corresponds to the concept of apical dominance, which we assume here.

The set $T_{n+1}^i \subset \mathbb{L}^{n+1}$ captures the unique topological history of bifurcations of order $n+1$ branches terminating at the leaf in x_i. For an order $n+1$ branch leading to $\mathbf{x_i}$ that bifurcates from a branch leading to $\mathbf{x_j}$, the first two indices of T_{n+1}^i are ij, etc. By construction we have $|G_n^i| = |T_n^i|$. Formally, we have