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A dynamical systems framework for crop models: Toward optimal fertilization and irrigation strategies under climatic variability

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ABSTRACT

Crop models are widely used for the modeling and prediction of crop yields, as decision support tools, and to develop research questions. Though typically constructed as a set of dynamical equations, crop models are not often analyzed from a specifically dynamical systems point of view, despite its potential to elucidate the roles of feedbacks and internal and external forcings on system stability and the optimization of control protocols (e.g., irrigation and fertilization). Here we develop a minimal dynamical system, based in part on the widely known AquaCrop model, consisting of a set of ordinary differential equations (ODE's) describing the evolution of canopy cover, soil moisture, and soil nitrogen. These state variables are coupled through canopy growth and senescence, the evapotranspiration and percolation of soil moisture, and the uptake and leaching of soil nitrogen. The system is driven by random hydroclimatic forcing. Important crop model responses, such as biomass and yield, are calculated, and optimal yield and profitability under differing climate scenarios, irrigation strategies, and fertilization strategies are examined within the developed framework. The results highlight the need to maintain the system at or above resource limitation thresholds to achieve optimality and the role of system variability in determining management strategies.

1. Introduction

As tools to forecast or backcast crop yields, improve management strategies, and better understand the physical processes underlying crop production, crop models are important tools from both a research and an engineering viewpoint (Wallach et al., 2006; Steduto et al., 2009). The model outputs, structure, parameterization, and data assimilation are all active areas of crop modeling research. Because different users have different goals, several types of crop models have been proposed, which can be categorized in a number of ways. One of the most basic distinctions is between dynamic crop models, which are comprised of a set of differential equations, which are then integrated in time to simulate the crop responses of interest at each time point (often daily), and crop response models, which, though they may be built on dynamic models, relate crop responses directly to inputs (Thornley and Johnson, 1990; Wallach et al., 2006). Most crop models have as their main state variables above-ground biomass, leaf area index (LAI), harvestable yield, and water and nitrogen balances, though the choice and precise number of state variables varies (Wallach et al., 2006). Virtually all crop models are process-based, but necessarily involve empirical components, and are of varying levels of complexity, depending on the particular goals of the model and on the availability of input data. Some are specific to certain crops or groups of crops, such as CERES (Ritchie et al., 1998) and AZODYN (Jeuffroy and Recous, 1999), while others are more generic, such as CROPGRO (Boote et al., 1998), CROPSYST (Stöckle et al., 2003), STICS (Brisson et al., 2003), and some focus on particular regions (e.g., INFOCROP (Aggarwal et al., 2006) for tropical regions). Also in the category of generic models, but with a more parsimonious framework, is AquaCrop (Steduto et al., 2009). Despite the abundance of crop models which have dynamical systems at their core, they are not often analyzed as dynamical systems *per se* – that is, using the wide array of tools and methods provided by dynamical systems theory to understand the mathematical behavior and properties of the models (Strogatz, 2014). There are a number of potential reasons for this, such as the difficulty of applying these

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methods to complex models and the aims of modelers, which may be focused toward other goals.

Although they tend to be considerably more complex and serve different purposes, crop models share many features and describe many of the same processes as do minimal ecohydrological models. The use of such models, which are typically formulated as dynamical systems, has provided many insights into soil moisture dynamics, plant–water interactions, and nutrient cycling (Rodríguez-Iturbe et al., 1999; Porporato et al., 2002, 2003; Rodríguez-Iturbe and Porporato, 2004). Some features of this type of ecohydrological model, such as the parsimonious representation of processes and stochastic and dynamic coupling between state variables, are well-suited to study the feedbacks, nonlinearities, and effect of random hydroclimatic forcing on agroecosystems (Porporato et al., 2015). Indeed, the underlying assumptions of many dynamic ecohydrological models are better met in agroecosystems than in the natural ecosystems where they are normally applied. Such assumptions include homogenous soil depth and plant spacing, as well as good drainage, which describe well an agricultural field with tillage, uniform crop spacing, and tile drains.

Various studies have used a dynamical systems framework to examine grass ecosystems (Thornley and Verberne, 1989; Tilman and Wedin, 1991), grass growth modulated by competition with legumes (Thornley et al., 1995) and grazing (Johnson and Parsons, 1985), forest ecosystems (Thornley and Cannell, 1992), forest ecosystems under harvest (Parolari and Porporato, 2016), soil salinity and sodicity (Mau and Porporato, 2015), and the cycles themselves, including feedbacks and nonlinearities (Porporato et al., 2003; Manzoni et al., 2004; Manzoni and Porporato, 2007). Studying crop models with dynamical systems theory allows for the more ready exploration of many interesting aspects of crop systems, including their stability with respect to parameter change, the feedbacks between water, carbon, and nutrient cycling, the optimal conditions for growth, and the impact of external inputs such as changes in climate patterns and management choices (i.e. fertilization and irrigation).

With the goal of taking advantage of the tools of dynamical systems theory, in this work we develop a dynamic crop model which captures the main crop fluxes and responses of interest without being overly complex. The model has three main variables which interact dynamically: the canopy cover, the relative soil moisture, and the soil nitrogen. The differential equations which account for these components are coupled via the crop growth, nitrogen uptake and leaching, and evapotranspiration terms. Biomass and yield, which are not considered to interact dynamically with the other state variables but rather are determined by them, are also included as derived variables of agroecologic interest. The model is used to examine the crop response to water and nutrient availability and varying climatic conditions in order to examine questions of optimal fertilization and irrigation and reduction of nutrient leaching.

Several aspects of the model are derived from AquaCrop (Steduto et al., 2009; Raes et al., 2009; Hsiao et al., 2009), which is the existing generic crop model that, in addition to its parsimony, can perhaps most easily be viewed as a dynamical system. It is also physically based, validated for a variety of crops, and widely known. AquaCrop itself is largely based on earlier FAO publications, in particular through its use of crop coefficients (Allen et al., 1998) and in the relation between crop water uptake and yield (Doorenbos and Kassam, 1998). The most notable similarities between the model developed here and AquaCrop are that canopy cover is used rather than the more typical LAI, that evapotranspiration is represented by crop coefficients, and in the dependence of the partitioning of transpiration and evaporation on the canopy cover. Some key differences involve the soil moisture balance (the model developed here makes use of a single vertically averaged soil moisture value rather than a soil column consisting of multiple layers, and it uses the same soil moisture stress thresholds throughout) and the nitrogen balance (a balance of total mineral nitrogen in the soil is used here rather than the empirical fertility coefficient employed in AquaCrop).

Here a different viewpoint and set of tools is emphasized for studying dynamic crop models, and we also aim to place crop models in a dynamical systems context and to discuss the application of the associated methods to crop models. We hope that this contribution will be of interest to both the crop modeling community and to researchers in the area of theoretical ecohydrology as a means to explore the response of agroecosystems to uncertain climatic conditions and optimal management strategies.

2. Model components

In this section a dynamical system is constructed which describes the interaction of three main components: canopy cover $C(t)$, relative soil moisture $S(t)$, and total nitrogen content in the soil $N(t)$. We also consider two related variables, namely the crop biomass $B(t)$ and the crop yield $Y(t)$ (hereafter we drop the t-dependence of the state variables). The model is interpreted at the daily timescale (no diurnal dynamics are considered) and applied over the course of a single growing season. It can be forced by random rainfall inputs (Rodríguez-Iturbe and Porporato, 2004), and is assumed to apply to an agricultural field which is homogenous in terms of soil composition, climatic forcing, and management.

2.1. Canopy cover dynamics

We define the canopy cover to be the fraction of ground covered by a crop. The benefit of using this alternative to the LAI, which was also employed by AquaCrop (Steduto et al., 2009), is that it combines multiple attributes of the crop canopy into a single, easily measured or estimated variable. The rate of change in canopy cover is modeled as a balance between the increase due to canopy growth and the decrease due to metabolic limitations and senescence, so that

$$\frac{dC}{dt} = G(C, S, N, t) - M(C, t), \tag{1}$$

where G is the canopy growth rate, and M is a term which combines the effects of metabolic limitation and senescence. The growth rate is assumed to be proportional to the rate of nitrogen uptake, U (discussed further in Section 2.3), giving

$$G(C, S, N, t) = r_G \cdot U(C, S, N, t), \tag{2}$$

where r_G is the canopy cover increase per amount of nitrogen taken up (the value for this and other crop growth parameters can be found in Table 1). The combined metabolic limitation and mortality/senescence term is

$$M(C, t) = (r_M + \gamma(t - t_{sen}) \cdot \Theta(t - t_{sen})) \cdot C^2, \tag{3}$$

where the first term, r_M, is a constant metabolic limitation term, and the next term is a time-dependent mortality and senescence term. For the latter, a linear function is used which increases with a slope of γ after the senescence onset time, t_{sen}, at which point the Heaviside step function, Θ, causes the senescence term to begin to affect the equation. This form recalls somewhat the Gompertz–Makeham law (Makeham, 1860), which includes an age-independent mortality term and an age-dependent mortality term, although here the constant term is conceptualized as a metabolic limitation term and the time-dependent term as a senescence term. For unstressed conditions (sufficiently high S and N) prior to t_{sen}, Eq. (1) is the logistic growth equation (Murray, 2002), and it includes the approximately exponential growth of C in the initial growth stage, the slowing of