



## Ecological utility theory: Solving a series convergence issue



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### ABSTRACT

Utility analysis (Patten, 1991; Fath and Patten, 1999) is quite useful in quantifying direct and indirect species relations in a compartmental ecosystems model, regardless of its size or complexity. It serves as a basis for the formulation of system-wide synergism (Fath and Patten, 1998) and mutualism (Fath, 2007) measures. A significant issue that limits the applicability of utility analysis is that its mathematical formulation requires the convergence of a matrix power series, which may fail for otherwise perfectly valid ecosystem models. For example, utility analysis for the well known Neuse river estuary nitrogen flow models (Baird and Ulanowicz, 1989), collected over 4 years (16 seasons total), do not converge for some seasons, but converge for others. Interestingly, ecologists find the analysis results meaningful and useful, even when the convergence criteria are not satisfied. This work investigates the cause for this divergence, analyzes the properties of the matrix power series, and uses an alternative summability method which transforms the diverging matrix power series into a converging one. In particular, we show that finitely many applications of the Euler transform are capable of forcing convergence on an otherwise diverging matrix power series for utility analysis. While the divergence in the regular sense remains, this work brings forward a strong mathematical argument that the utility analysis, synergism and mutualism indices, are useful for all ecological network models, regardless of their convergence characteristics.

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### 1. Introduction

Utility analysis (Patten, 1991; Fath and Patten, 1999) is an invaluable tool to assess the harm or benefit of a species to another, a species to its ecosystem, and the total sum of harm and benefit experienced by the entire ecosystem. The latter system-wide measure is called the synergism index (Fath and Patten, 1998). Utility analysis applies to flow network models of conservative quantities (energy, matter), often depicted as directed graphs. These consist of  $n$  compartments (nodes, vertices) interconnected by a set of directed flows (directed links, edges). The compartments denote standing stocks ( $x$ ) as storages of the energy or matter, such as the

total biomass of a certain species living in an area. These quantities are transferred as directed flows ( $F_{ij}$ ) between compartment pairs.

$F_{ij}$  : Flow rate from compartment  $j$  to compartment  $i$

$z_i$  : Environmental input rate into compartment  $i$

$y_i$  : Environmental output rate from compartment  $i$

$x_i$  : Storage amount at compartment  $i$

(1)

Natural systems are composed of thousands, or even millions, of individuals interacting while the compartments and flows are idealized simplifications of these interactions attempting to model the overall fluxes of the studied quantity between different modes of residence within the system. The network “flows” considered in ecological models are point transfers of mass or energy between the node storages, representing interactions such as feeding among species. The transfer set so constructed represents a system-of-definition, open to energy and matter exchange at the system boundaries, incoming as inputs ( $z$ ), outgoing as outputs ( $y$ ). The inputs and storages generate the flows out from a compartment,

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whose sums at  $i$ th nodes are the outgoing throughflows ( $T_i^{out}$ ).

$$T_i^{out} = y_i + \sum_{j=1}^n F_{ji}$$

$$T_i^{in} = z_i + \sum_{j=1}^n F_{ij}$$

The total rate of matter or energy received by a compartment defines the incoming throughflow ( $T_i^{in}$ ). The difference of the incoming and outgoing throughflows defines the change in storage, forming a differential equation

$$\frac{dx_i}{dt} = T_i^{in}(t) - T_i^{out}(t). \tag{2}$$

If the storage values stay constant over time ( $dx_i/dt=0$ ), meaning that the system is at steady-state, then the incoming and outgoing throughflows for each compartment are equal to each other ( $T_i^{in} = T_i^{out} = T_i$ ). One advantage of utility analysis is that it does not require any information about flow kinetics or dynamics. For an ecosystem model represented as a differential equation (2), it is relatively easy to perform perturbation simulations to measure the effects of a compartment on others. However, deriving an accurate differential equation model of an ecosystem is no easy task, and sometimes not even feasible. Utility analysis quantifies compartmental relationships using only flow rates among compartments and the environment at steady state.

Utility analysis is built on the *direct utility matrix*  $D$  (Patten, 1991), defined as

$$D_{ij} = \frac{F_{ij}}{T_i} - \frac{F_{ji}}{T_j} \tag{3}$$

utilizing the steady state assumption that total input into and total output from each compartment equal each other.  $D_{ij}$  quantifies the relative benefit ( $D_{ij} > 0$ ) or harm ( $D_{ij} < 0$ ) done by compartment  $j$  to compartment  $i$ , based on only the direct interactions. For instance, if  $j$  consumes  $i$ ,  $F_{ji} > 0$  and clearly compartment  $j$  is harmful for  $i$ , but the relative intensity of this harm depends on the existence of other consumers of  $i$ . For example, if  $j$  is the only consumer of  $i$ , then  $T_i = F_{ji} + y_i$  so  $D_{ij} = -F_{ji}/(F_{ji} + y_i)$ , is near to  $-1$ , indicating that  $j$  does a lot of the harm to  $i$ . But if compartment  $i$  has multiple consumers,  $T_i$  will be larger and the relative harm to compartment  $i$  done by compartment  $j$  will decrease. Hence, the ratio  $-F_{ji}/T_i$  represents the relative harm  $j$  does to  $i$ . Similarly, if  $i$  consumes  $j$  ( $F_{ij} > 0$ ), then  $j$  is beneficial for  $i$ . The ratio  $F_{ij}/T_i$  represents how beneficial  $j$  is for  $i$ , among all resources of  $i$ . Eq. (3) defines  $D$  as a sum of this direct benefit and harm received by  $i$  from  $j$ , in other words, the direct utility of  $j$  for  $i$ .

The *utility analysis matrix*  $U_{ij}$  quantifies how beneficial ( $U_{ij} > 0$ ) or harmful ( $U_{ij} < 0$ ) compartment  $j$  is for  $i$  over all possible connections, direct and indirect. Second order effects of  $j$  on  $i$  are given by the  $ij$  entry of the squared matrix,  $D^2$ . Indeed, the  $ij$  coefficient of  $D^2$  is given by  $(D^2)_{ij} = \sum_k D_{ik}D_{kj}$  with  $D_{ik}D_{kj}$  being the product of the relative good (or harm) done by compartment  $j$  to compartment  $k$  with the relative good (or harm) done by compartment  $k$  to compartment  $i$ . Summing over all compartments  $k$  gives the total second order effects of compartment  $j$  on compartment  $i$ . Similarly, all  $n$ th order effects are given by the elements of the  $n$ th power,  $D^n$ . Therefore  $U$  is defined as a matrix power series of the  $D$  matrix, similar to the definitions of pathway, throughflow, and storage analyses (Patten, 1978, 1985; Fath and Patten, 1999):

$$U := I + \underbrace{D}_{\text{Direct}} + \underbrace{D^2 + D^3 + \dots}_{\text{Indirect}} \tag{4}$$

**Table 1**

Computations for utility analysis for the two models shown in Fig. 1. The numerical results in this table confirm the visual results presented in Fig. 1(c) and (d). The information on the first column clearly shows that  $1 + D + \dots + D^n$  converges to  $(I - D)^{-1}$  as  $n \rightarrow \infty$  for Model (a), whereas the second column shows that such convergence is not valid for Model (b), and we have  $1 + D + D^2 \dots \neq (I - D)^{-1}$ .

	Model (a)	Model (b)
$D$	$\begin{bmatrix} 0 & -0.51 & -0.24 \\ 1 & 0 & -0.52 \\ 0.48 & 0.52 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -0.59 & -0.09 \\ 1 & 0 & -0.66 \\ 0.34 & 0.66 & 0 \end{bmatrix}$
$\sum_{m=0}^{25} D^m$	$\begin{bmatrix} 0.77 & -0.39 & -0.06 \\ 0.57 & 0.71 & -0.44 \\ 0.51 & 0.26 & 0.85 \end{bmatrix}$	$\begin{bmatrix} 1.59 & -1.05 & -0.75 \\ 2.17 & 1.91 & -1.11 \\ 0.10 & 1.47 & 1.44 \end{bmatrix}$
$\sum_{m=0}^{100} D^m$	$\begin{bmatrix} 0.67 & -0.33 & 0.01 \\ 0.39 & 0.59 & -0.40 \\ 0.52 & 0.15 & 0.80 \end{bmatrix}$	$\begin{bmatrix} 25.9 & 27.2 & -6.9 \\ -29.1 & 39.3 & 32.3 \\ -37.5 & -16.9 & 19.75 \end{bmatrix}$
$(I - D)^{-1}$	$\begin{bmatrix} 0.67 & -0.34 & 0.01 \\ 0.39 & 0.59 & -0.40 \\ 0.53 & 0.15 & 0.79 \end{bmatrix}$	$\begin{bmatrix} 0.68 & -0.35 & 0.09 \\ 0.37 & 0.51 & -0.41 \\ 0.48 & 0.22 & 0.76 \end{bmatrix}$
Eigenvalues of $D$	$\begin{bmatrix} 0 \\ -0.95i \\ +0.95i \end{bmatrix}$	$\begin{bmatrix} 0 \\ -1.045i \\ +1.045i \end{bmatrix}$

Since  $(D^m)_{ij}$  represents the harm and/or benefit received by  $i$  from  $j$  over all paths of length  $m$ ,  $U$ , defined as the sum of all powers of  $D$ , represents the relationship among all compartments, taking into account all direct and indirect connections.

## 2. Occasional failure of utility analysis computations

A significant problem with the mathematical formulation of the utility matrix (4) limits its use. A necessary condition for the infinite sum of the powers of  $D$  to converge to a finite value is that the elements in the infinite sum must become smaller (converge to zero), or at least partially cancel out. In certain cases, the elements of the matrix  $D^m$  may alternate between increasingly high positive and negative values as the matrix power  $m$  increases, as shown in Fig. 1(d). For those cases the sum defining  $U$  diverges. If the infinite sum converges, it must converge to the matrix  $(I - D)^{-1}$ . This matrix can be constructed regardless of the convergence of the infinite sum (Fath, 2004). It is perhaps tempting to simply define the utility matrix  $U$  to be  $(I - D)^{-1}$  but then the original motivation of summing all higher order effects is lost.

Indeed, most software performing utility analysis, such as EcoNet (Kazanci, 2007; Schramski et al., 2011), enaR (Borrett and Lau, 2014) and NEA.m (Fath and Borrett, 2006), naturally use  $(I - D)^{-1}$  to compute  $U$ , as it is not feasible to compute an infinite sum of matrix powers. So the software may display a utility matrix even in the event that the sum defining utility diverges. The equivalence  $(I - D)^{-1} = \sum_{m=0}^{\infty} D^m$  relies on an apparently fragile limit operation that may fail for some models. Unfortunately no clear ecological reason has been provided for this failure in the literature so far. For example, the well known Neuse river estuary nitrogen flow models (Baird and Ulanowicz, 1989) contain 16 ecological network models based on data collected for four seasons over four years. The utility analysis matrix converges for some seasons, but not for others. Nevertheless,  $(I - D)^{-1}$  can be computed for all seasons, and appears to provide reasonable and meaningful information. Yet, without the necessary convergence, we have no clear explanation as to what the matrix  $(I - D)^{-1}$  represents.

To investigate the issue further, we built two similar models with identical network structures, but with slightly different flow values, shown in Fig. 1(a) and (b). Table 1 shows essential matrices computed for these two models. The results shown in Fig. 1(c), (d) and Table 1 clearly demonstrates that the convergence criterion is satisfied by Model (a), but not by Model (b), despite the strong similarity between the two models. Current methodology limits the application of utility analysis to Model (a), and obtained results

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