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Truncated Gaussian and derived methods

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ABSTRACT

The interest of a digital model to represent the geological characteristics of the field is well established. However, the way to obtain it is not straightforward because this translation is necessarily a simplification of the actual field. This paper describes a stochastic model called truncated Gaussian simulations (TGS), which distributes a collection of facies or lithotypes over an area of interest. This method is based on facies proportions, spatial distribution and relationships, which can be easily tuned to produce numerous different textures. Initially developed for ordered facies, this model has been extended to complex organizations, where facies are not sequentially ordered. This method called pluri-Gaussian simulation (PGS) considers several Gaussian random functions, which can be correlated. PGS can produce a large variety of lithotype setups, as illustrated by several examples such as oriented deposits or high frequency layering.

1. Introduction

The conversion of the geological characteristics of a field into a digital model is necessary to perform analyses such as scheduling the exploitation of a mine or detecting the pollution in a sedimentary environment. This conversion can be performed in two main different manners: either by understanding and mimicking the sedimentary processes (genetic models) or by directly reproducing the current facies arrangement resulting from several sedimentary and transformation processes (stochastic models).

The genetic models are the most efficient way to reproduce realistic sedimentation textures as they are based on physical processes (Cojan et al., 2005), but they require the precise knowledge of the whole set of processes that have led to the studied deposit. Moreover, honoring exactly the information provided by the data is still challenging (conditioning step). On the contrary, for stochastic models which are based on the resulting actual deposit image, the conditioning step is usually tractable. Different stochastic models are classically used; in those methods the texture characteristics are provided either in the training image for the multipoint simulation (MPS) (Mariethoz et al., 2010; Strebelle, 2002) or through the multivariable stochastic model for the sequential indicator simulation (SIS) (Alabert, 1987; Emery, 2004) and the truncated Gaussian model (TGS) (Matheron et al., 1987). With the latter TGS method, it is easy to define a lot of different multivariate models and hence to produce a large variety of arrangements with different relationships between facies.

The basic ingredients of TGS consist in the proportions of the facies and their spatial distribution and relationships. Initially the truncated Gaussian model has been introduced for reproducing a simple organization of ordered lithotypes. The lithotypes are then obtained by thresholding a single underlying Gaussian random function (GRF). It suffices to split the total domain of variation

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of the GRF in intervals and to assign each interval to a lithotype. The bounds of the interval (or the thresholds of the underlying GRF) are calculated so as to match the proportions of the various lithotypes. Finally the spatial characteristics of the GRF are related to those of the lithotype indicators which are described by their experimental variograms.

When the lithotype organization is more complex, in particular not sequentially ordered, it is necessary to consider several GRFs (hence the method called pluri-Gaussian, PGS). In that case, each lithotype is defined by its thresholds along each GRF. The partition scheme of the different GRFs into lithotypes is described by a synthetic graph called the lithotype rule. Finally, if some lithotypes must present linked shapes, it is possible to introduce some dependency between the GRFs (correlating them for example).

This paper provides a detailed description of the truncated Gaussian simulations and their derived methods. By a series of examples, it illustrates the large variety of lithotype setups produced simply by varying the lithotype rule and/or using particular structures for the underlying GRFs.

2. Method description

The geological interpretation to be modeled is composed of different sets of interest which constitute a partition of the space. These sets are the qualitative variables (lithotypes or facies in this paper) to be reproduced.

2.1. Qualitative properties

In order to perform calculations with qualitative properties it is necessary to transform them into numerical values beforehand. This can be done using the indicator function: the indicator of a given lithotype is equal to 1 when the observed point belongs to this lithotype and 0 otherwise. There are as many indicators as there are lithotypes involved in the deposit description, which turns the qualitative properties into a multivariable numerical setup. Moreover, as the indicators are numerical variables, it is now possible to consider their spatial characteristics through traditional tools such as simple and cross-variograms. The indicator variogram measures the probability that two points do not belong to the same lithotype (Eq. 1) as a function of their distance:

$$\gamma_B(h) = 0.5[P(x \in B, (x+h) \notin B) + P(x \notin B, (x+h) \in B)] \quad (1)$$

where B is a lithotype, P a probability, x a point in the field, and h the distance between point x and another point in the field.

The indicator cross-variogram measures the probability that two points belong to two different lithotypes (Eq. 2) as a function of their distance:

$$\begin{aligned}\gamma_{AB}(h) &= 0.5E[(I_A(x+h) - I_A(x))(I_B(x+h) - I_B(x))] \\ &= -0.5[P(x \in A, (x+h) \in B) + P(x \in B, (x+h) \in A)]\end{aligned} \quad (2)$$

where A and B are two lithotypes and I their indicators, E the means, P a probability, x a point in the field, and h the distance between point x and another point in the field.

As the lithotypes constitute a partition of the space, when the indicator of a given lithotype is equal to 1, the indicators of the other lithotypes are equal to 0. This leads to particular relationships relating simple and cross-variograms of the indicators.

For instance, in Fig. 1a, a simulation has been performed using an object based model (Boolean simulation): in a white background, elongated grey objects are overlaid by black circular objects. Fig. 1b represents the simple variogram of each indicator calculated in north–south

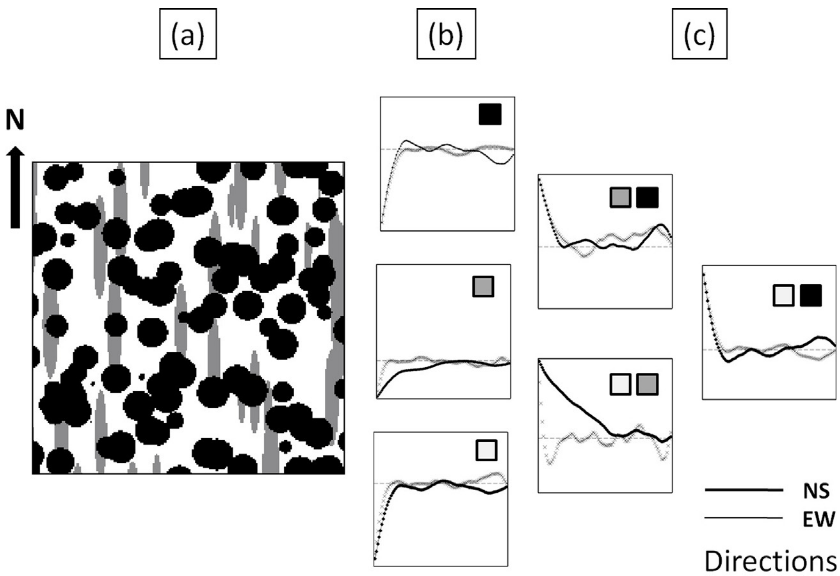

Fig. 1. On a simulated image in three sets (a), two directional variograms (NS and EW) are computed for each set (b: simple variograms) and for each pair of sets (c: cross-variograms).

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