C. R. Geoscience xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Comptes Rendus Geoscience

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Internal Geophysics

Local gravity field modeling using spherical radial basis functions and a genetic algorithm

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ARTICLE INFO

Article history:
Received 6 February 2017
Accepted after revision 4 March 2017
Available online xxx

Handled by Philippe Cardin

Keywords:
Radial Basis Functions
Local gravity field
Genetic algorithm
Conjugate gradient
Least squares

ABSTRACT

Spherical Radial Basis Functions (SRBFs) can express the local gravity field model of the Earth if they are parameterized optimally on or below the Bjerhammar sphere. This parameterization is generally defined as the shape of the base functions, their number, center locations, bandwidths, and scale coefficients. The number/location and bandwidths of the base functions are the most important parameters for accurately representing the gravity field; once they are determined, the scale coefficients can then be computed accordingly. In this study, the point-mass kernel, as the simplest shape of SRBFs, is chosen to evaluate the synthesized free-air gravity anomalies over the rough area in Auvergne and GNSS/Leveling points (synthetic height anomalies) are used to validate the results. A two-step automatic approach is proposed to determine the optimum distribution of the base functions. First, the location of the base functions and their bandwidths are found using the genetic algorithm; second, the conjugate gradient least squares method is employed to estimate the scale coefficients. The proposed methodology shows promising results. On the one hand, when using the genetic algorithm, the base functions do not need to be set to a regular grid and they can move according to the roughness of topography. In this way, the models meet the desired accuracy with a low number of base functions. On the other hand, the conjugate gradient method removes the bias between derived quasigeoid heights from the model and from the GNSS/leveling points; this means there is no need for a corrector surface. The numerical test on the area of interest revealed an RMS of 0.48 mGal for the differences between predicted and observed gravity anomalies, and a corresponding 9 cm for the differences in GNSS/leveling points.

1. Introduction

The series of spherical harmonics is the commonly used method for global gravity field modeling; however, Spherical Radial Basis Functions (SRBFs) can locally represent the gravity field of the Earth. The Stokes coefficients of spherical harmonics are sensitive to local signal changes, although SRBFs are often used to model the higher frequencies of the field and can be used as an alternative method for regional modeling of the Earth's gravity field and the corresponding quasigeoid models. The Earth's gravity field can be expressed by a linear combination of the SRBFs (Barthelmes and Dietrich, 1991). The kernels of SRBFs are mostly of the inverse-distance type, which can be defined in different ways; for instance, the point-mass kernel (Barthelmes, 1988;

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http://dx.doi.org/10.1016/j.crte.2017.03.001

Barthelmes and Dietrich, 1991; Blaha et al., 1986; Lin et al., 2014; Shahbazi et al., 2015; Sünkel, 1981; Weightman, 1965). Later the higher order of point-mass, radial multipoles, were introduced by Marchenko (1998) and used by Foroughi and Tenzer (2014). Poisson wavelets, were derived by Holschneider et al. (2003) and later used by Chambodut et al. (2005); Klees and Wittwer (2007); Klees et al. (2008); Panet et al. (2006); Safari et al. (2014); and Tenzer et al. (2012). The band-limited Blakman base functions were first used by Schmidt et al. (2004, 2005). Tenzer and Klees (2008) showed that there is no significant difference between different types of kernels when they are parameterized properly.

The parameterization of the gravity field using SRBFs consists in defining the type of kernel, the number of kernels, the location of the center of the kernels, and kernel bandwidths that all have a considerable effect on the predicted model. Using many kernels leads to overparameterization, while applying a low number of them can only represent the low-frequency part of the model. The SRBF kernel is defined on or within the sphere which is completely located inside the Earth's topography and is known as the Bjerhammar sphere (Moritz, 1980). SRBF kernels are typically defined as inverse distances between integration points in the target area and computation points in the data coverage. An inverse relation exists between the kernel's bandwidth and the kernel's depth into the Bjerhammar sphere. Some studies (Klees et al., 2008; Tenzer et al., 2012) consider the depth of the kernels as separate unknown parameters rather than their 2D location. However, the unknown parameters can be merged into 3D positions of the kernels on the Bjerhammar sphere and be found in one step (Shahbazi et al., 2016).

Finding the number of base functions (kernels) and the location of their centers is the first step of SRBF parameterization. Different methods have been proposed to find the optimum number with respect to the location of the observation points and the depth of kernels into the Bjerhammar sphere. Marchenko (1998) used radial multipole kernels below the observation points and then optimized the solution using a sequential multi-pole algorithm. The depth and order of the radial multi-pole kernels were then determined by the covariance function of the signal around the observation points. Klees and Wittwer (2007) and Klees et al. (2008) proposed a methodology that was fully based on the distribution of the data. They used an initial regular grid of the SRBFs to carry out the first adjustment; then they added local base functions to the areas where the residuals between observation and predicted anomalies were larger than a predefined tolerance. The generalized cross-validation method was also applied to approximate the optimal depth. Their method was only applicable in areas with dense gravity data coverage and smooth topography. Tenzer et al. (2012) analyzed the least squares approximation of the gravity field via Poisson wavelets of order 3 on various spherical equiangular grids, and they used the method of minimization of the RMS differences between predicted and observed gravity disturbances to find the optimal depth of the kernels. They reduced the number of the required base functions by applying topographical corrections. Foroughi and Tenzer (2014) introduced the Levenberg–Marquardt algorithm to minimize the number of base functions and find their depth based on Least Square (LS) adjustment; they suggested the use of a two-step process to find the optimal number of base functions. In the first step, the optimum number of base functions was defined based on the fitting between observed and predicted gravity anomalies and in the second step the optimum number was chosen according to best fitting between observed and predicted quasigeoidal heights. The common optimum number between the two steps was then selected as the number of kernels. The two-step method did not require adding extra local kernels manually, but it was computationally expensive and needed independent control points with two types of observations: gravity anomalies and normal heights. Later Shahbazi et al. (2016) modified this method so that the number of base functions could be chosen in one step. Their method allows one to find the optimum number based on estimated errors in the observation data as well as their distribution.

The proposed methods of parameterization of SRBFs in previous studies are limited to the initial choice of the location and depth of the kernels and some are not able to choose the number of kernels automatically. These methods might represent the "*local minimum*" of the parameterized solution because their final solution does not differ significantly from their initial values. They mostly use corrector surfaces to fit the final gravity model to local GNSS/Leveling data points, which simply hides the discrepancies between the predicted and the observed model. The intention of the current study is to employ the *Genetic Algorithm*" (GA) and let the parameters of the kernel of SRBFs be chosen based on the information provided in the observation data. The proposed method in this study can search among all the possible solutions of parameterized SRBFs and find the "*global minimum*" of the target function that is set to be minimized in the process. Once the parameters of SRBFs are found using GA, the system of linear equation, in the LS sense, is solved based on the Conjugate-Gradient (CG) technique, which leads to an un-biased solution, unlike the previously used approaches.

The theory of SRBFs in gravity field modeling is described in Section 2. General information on GA is provided in Section 3.1. The iterative approach of CG is presented in Section 3.2. The methodology of problem solving is explained in Section 4. The numerical results and discussion are touched on in Section 5 and Section 6, respectively. At the end, Section 7 summarizes the remarks of this contribution.

2. Theory of regional gravity field modeling using SRBFs

According to the Runge–Krarup theorem, a harmonic function can be regarded as an expansion of the non-orthogonal base functions. The disturbing potential of the Earth's gravity field is considered harmonic above the geoid (Moritz, 1980), therefore, we can represent it as a linear combination of the set of non-orthogonal base