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Modeling spreading of oil slicks based on random walk methods and Voronoi diagrams

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ABSTRACT

We introduce a methodology for representation of a surface oil slick using a Voronoi diagram updated at each time step. The Voronoi cells scale the Gaussian random walk procedure representing the spreading process by individual particle stepping. The step length of stochastically moving particles is based on a theoretical model of the spreading process, establishing a relationship between the step length of diffusive spreading and the thickness of the slick at the particle locations. The Voronoi tessellation provides the areal extent of the slick particles and in turn the thicknesses of the slick and the diffusive-type spreading length for all particles. The algorithm successfully simulates the spreading process and results show very good agreement with the analytical solution. Moreover, the results are robust for a wide range of values for computational time step and total number of particles.

1. Introduction

Oil spilled at sea typically forms a slick at the air-water interface, and begins immediately to spread out. It is fundamentally important to estimate how the oil slick will spread and move on the surface of sea because the spreading and transport processes interact with the processes of evaporation, dissolution, weathering, and entrainment, which in turn affect the eventual fate and effects of the spilled oil. Spreading models have therefore received attention by scientists and researchers over the years (e.g. Fay, 1969; Hoult, 1972; Mackay et al., 1980a, 1980b; Nihoul, 1984; Elliott et al., 1986; Korinenko and Malinovsky, 2014). A comprehensive review of such models is presented by Reed et al. (1999). The empirical model described in Fay (1969) is worth mentioning because it was one of the earliest and the concept and equation developed by Fay are still applied in many of today's spreading algorithms. Later on, several modifications, enhancements or improvements on the empirical spreading models have appeared in the literature (e.g. Hoult, 1972; Mackay et al., 1980a, 1980b; Lehr et al., 1984; Johansen, 1984; Venkatesh et al., 1990; Elliott et al., 1986). With the advent of increasing computational power, it has become possible to simulate the spreading of oil slicks by solving the partial differential equations describing the process even in complex problem domains. The numerical solutions can be classified into two major groups: discretizing the governing equations in fixed space (the Eulerian approach), and approximating the solution field with particles moving stochastically (the Lagrangian approach). Models described in Tkalich et al. (2003), Di Martino and Peybernes (2007) are examples of Eulerian models. This type of model solves numerically the coupled mass and momentum balance equations accompanied with empirical parametrizations. On the other hand, Lagrangian models appear more in the literature (e.g. Al-Rabeh et al., 1989; Zhang et al., 1997; Cekirge et al., 1997; Arkhipov et al., 2008; and Guo et al., 2014) because they can simulate complexities such as irregular and discontinuous oil slick shapes more easily than fixed grid Eulerian models.

The use of Voronoi (or Thiessen) polygons and triangular meshes for the slick representation has been mentioned and investigated by Galt (1994, and personal communication) and Lehr (2002, and personal communication). However, neither presents a methodology or discusses relevant results in detail. Delaunay triangulation, which is indeed the dual graph of the Voronoi diagram could also have been used, but restricts the possible solutions to simple triangles (e.g. Victoria et al., 2011). Moreover, Delaunay triangulation constructs a convex envelope for a set of points in a plane. Therefore it may not be suitable for non-convex and irregular slick shapes.

In the present paper, we address the spreading of spilled oil on the sea surface. An algorithm that represents the oil slick with a Voronoi diagram simulates the spreading process. The algorithm employs the Gaussian random walk method, representing the spreading process through individual particle stepping. Calculation of the step length for the diffusive spreading is based on a theoretical model of the spreading process rather than empirical parametrizations which are often chosen in the literature.

2. Material and methods

Transport and spreading of oil slicks on the sea surface is a complex phenomenon involving advection, buoyancy and surface tension forces,

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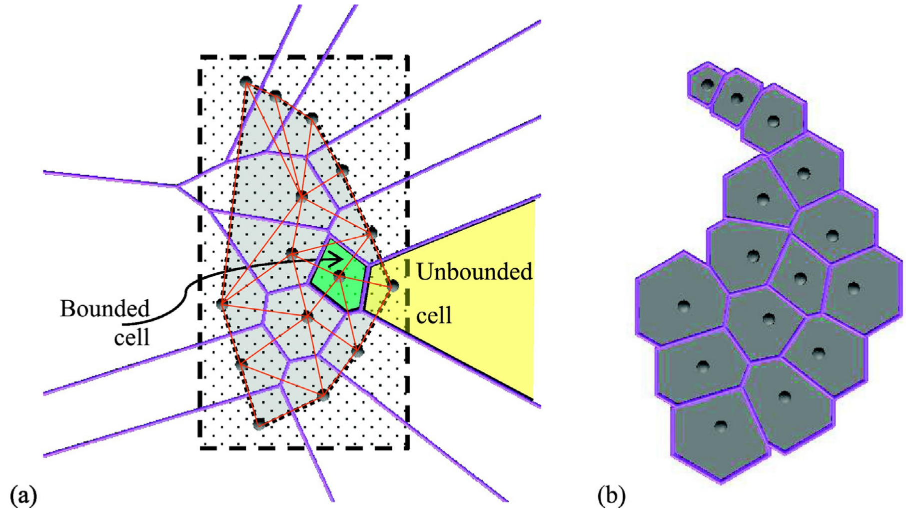

Fig. 1. (a) Voronoi diagram, Delaunay triangulation and convex hull of a set of slick particles computed by a conventional algorithm showing bounded and unbounded cells, (b) Voronoi diagram computed by the Voro++ library.

plus changing oil properties simultaneously acting on oil. Nihoul (1984) presented a model to describe the transport and spreading of oil slick which may be written:

$$\frac{\partial h}{\partial t}+\nabla\cdot(hu)=Q+\nabla\cdot\left(\frac{(\rho_w-\rho_o)}{\rho_w}g\frac{\rho_o}{k}h^2\nabla h-\frac{\gamma}{k}h\nabla\psi^2\right), \tag{1}$$

where h(m) is the slick thickness, u(m/s) is the wind- and current-induced advective velocity of the oil slick averaged over the slick thickness, t(s) is time, ρ_w and ρ_o(kg/m^3) are the water and oil densities, g(m/s^2) is the gravitational acceleration, γ(kg/s^2) is the surface tension, k(kg/m^{2}s) is the drag coefficient. The function ψ reflects the variation of surface tension changing from 0 at the center to 1 at the edges of the slick. Q(m/s) is a source-sink term representing continued oil release or loss due to entrainment from the surface slick into the water column or evaporation of oil to the atmosphere.

The above equation describes spreading of an oil slick, taking into account the gravitational, viscous and surface tension forces but neglecting the inertial force. Based on the early work by Fay (1969) and Hoult (1972), the traditional view is that the spreading process proceeds in three phases; the gravity-inertial spreading, the gravity-viscous spreading and the surface tension-viscous spreading. The first phase lasts only a few minutes depending on the size of slick, while the last one corresponds to the dispersion and separation phase of the slick. Many models consider primarily gravity-viscous spreading (Reed et al., 1999). If we therefore drop the surface tension term from the Nihoul model, consider only an instantaneous release of oil, and neglect loss processes, we obtain:

$$\frac{\partial h}{\partial t}+\nabla\cdot(hu)=\nabla\cdot(D(h)\nabla h) \tag{2}$$

The coefficient of diffusive spreading $D(h)$ is defined as follows:

$$D(h)=ah^2, a=\frac{g'\rho_o}{k}, g'=\frac{(\rho_w-\rho_o)}{\rho_w}g, \tag{3}$$

where g' is the reduced gravity.

In addition, when the source/loss term Q is neglected and if the variation of the advection velocity is assumed to be constant over the slick area, the slick can be considered to spread in a moving coordinate system with origin at its center of mass. If it is further assumed that the slick is axisymmetric, one can write the spreading equation in radial coordinates:

$$\frac{\partial h}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD(h)\frac{\partial h}{\partial r}\right), \tag{4}$$

where r(m) is the radial distance from the center of the slick. Nihoul (1984) proposed a self-similarity solution to this equation as a function of the radial distance r for the slick thickness having the form of:

$$h=\frac{3V}{2\pi R^2}\left(1-\frac{r^2}{R^2}\right)^{1/2}. \tag{5}$$

Here R(m) is the slick radius, and V(m^3) is the volume of the released oil. Nihoul (1984) calculated the slick radius for the gravity-viscous phase of spreading as follows:

$$R=\left(\frac{27V^2}{2\pi}at\right)^{1/6}, \tag{6}$$

which is the asymptotic solution to Eq. (4) with the self-similarity form of Eq. (5).

On the other hand, the spreading process can also be simulated with numerical methods, such as finite difference, finite element and discrete particle tracking. Eq. (2) describing the spreading of the oil slick is analogous to the advection-diffusion equation (ADE).

The ADEs can be solved by a discrete particle tracking algorithm employing the Gaussian random walk approach. For the two dimensional problem, the Gaussian random walk method moves a particle from its current position at (x_i^t,y_i^t) at time t to a new position after a time step of Δt with the following position update equations:

$$\begin{aligned}x_i^{t+\Delta t}&=x_i^t+u_x\Delta t+\mathcal{N}(\xi)\sigma_x,\\ y_i^{t+\Delta t}&=y_i^t+u_y\Delta t+\mathcal{N}(\xi)\sigma_y,\end{aligned} \tag{7}$$

where $\mathcal{N}(\xi)$ is a random number from the standard normal (Gaussian) distribution (μ=0, σ^2=1), σ_x and σ_y are the diffusion lengths in x and y directions. For the isotropic case, $\sigma_x=\sigma_y=\sigma=\sqrt{2D\Delta t}$, where D is the diffusion coefficient.

Arkhipov et al. (2008) made an analogy between the spreading and diffusion processes, and defined the length of the diffusive spreading as:

$$\sigma=\sqrt{2\lambda D(h)\Delta t}. \tag{8}$$