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Punishment in the form of shared cost promotes altruism in the cooperative dilemma games

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ABSTRACT

One phenomenon or social institution often observed in multi-agent interactions is the altruistic punishment, i.e. the punishment of unfair behavior by others at a personal cost. Inspired by the works focusing on punishment and the intricate mechanism behind it, we theoretically study the strategy evolution in the framework of two-strategy game models with the punishment on defectors, moreover, the cost of punishing will be evenly shared among the cooperators. Theoretical computations suggest that larger punishment on defectors or smaller punishment cost incurred by cooperators will enhance the fixation of altruistic cooperation in the population. Through the replicate dynamics, the group size of the randomly selected individuals from the sufficiently large population will notably affect the strategy evolution in populations nested within a dilemma. By theoretical modeling the concept of shared cost for punishment from one point of view, our findings underscore the importance of punishment with shared cost as a factor in real-life decisions in an evolutionary game context.

1. Introduction

Cooperative dilemmas describe the situations in which the optimal decision of an individual may conflict with the optimal decision for the group. Such situations are commonplace in the real world and attract the attention of researchers from multiple areas, since their successful solution can give us much hints in resolving the cooperative dilemma (Hoffman et al., 2015; Gallo and Yan, 2015). A standard framework utilized to investigating this problem is evolutionary game theory (Li and Kendall, 2014; Gokhale and Traulsen, 2010; Nowak, 2006). Paradigmatic examples of such dilemmas widely applied to the study of cooperative phenomena include the Prisoner's Dilemma game (PDG), which constitutes powerful metaphors to describe conflicting situations often encountered in natural and social sciences (Frean, 1994; Greig and Travisano, 2004; Hilbe et al., 2014).

Investigating the origin of cooperation among selfish individuals has aroused a variety of interesting proposed explanations. Previous studies have proposed or found some crucial factors for the evolution of altruistic traits, for example, kinship (Krupp et al., 2008), volunteering (Hauert et al., 2002; Semmann et al., 2003; Hauert and Szabó, 2003), partner selection (Coricelli et al., 2004), obligations (Galbiati and Vertova, 2008), group size (Carpenter, 2007; Janssen and Goldstone, 2006). Further, the evolution of cooperative behavior may be dependant on certain environmental conditions. One such condition that has been extensively studied is the introduction of a spatially structured population (Nowak and May, 1992; da Silva, 2008; Zhang et al., 2010; Gross and Blasius, 2008; Perc and Szolnoki, 2010). Moreover, the evolution of cooperation is also possible with a novel model of a population of agents that can move between groups (Zhang et al., 2011). Besides, other mechanisms are also making contributions in trying to crack this cooperative dilemma, such as peacemakers (Halevy and Halali, 2015), hierarchy (Cronin et al., 2014), extortion strategies (Hilbe et al., 2013), the shadow of the future (Blake et al., 2015). Also, there is a growing literature on other strategy selections besides cooperation and defection, incurred by different choices in real social systems. These new roles include the loners (Castro and Toro, 2008), punishers (Dreber et al., 2008), or insured players (Zhang et al., 2013). And, some works (My and Chalvignac, 2010) have supported a noticeable effect of an attractive exit option on contribution levels. Studies on this topic show that voluntary participation can induce a recovery of cooperation levels when the payoff yielded by the exit option is high enough.

As an easy-to-understand mechanism commonly existing in real social systems, punishment may also yield a solution to the problem of cooperative dilemma. Considering the potential cost, one phenomenon often observed in social interactions is altruistic punishment, i.e. the

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punishment on unfair behaviors by others at a personal cost. From the perspective of gaining more benefits, this is puzzling because natural selection is against those who choose the costly punishment and in favor of those who free ride. Inspired by this idea, altruistic punishment has been proposed to resolve the perplexing problem of collective dilemma (de Quervain et al., 2004; Egas and Riedl, 2008; Janssen and Bushman, 2008). The study of Boyd et al. (2003) suggests that an asymmetry between altruistic cooperation and altruistic punishment allows altruistic punishment to evolve in populations who participate in one-time and anonymous interactions. The work of Chen et al. (2014) shows that sharing the responsibility to sanction defectors rather than relying on certain individuals to do so permanently can provide solutions for the problem of costly punishment. Moreover, Szolnoki et al. (2011) studies the strategy competition among unconditional defectors, cooperators, and cooperating pool punishers in spatial public goods games. A preceding study (Perc, 2012) shows that pool-punishment in structured populations can be sustainable, if second-order free-riders are sanctioned as well, and to a such degree that they have no chance to prevail. Different with the unconditional punishers who always impose the same fines on defectors, Szolnoki and Perc (2013) studies the conditional punishers who do so proportionally with the number of other punishers in the group. And, the gained phase diagrams in dependence on the punishment fine and cost reveal that the two types of punishers cannot coexist Helbing et al. (2010).

Besides, Nelissen (2008) performs two works to investigate the reputation-based accounts of altruism, which predict that the more players sacrifice to help others, the greater their ensuing benefits. They show that only altruists who invest most in the punishment of unfairness are preferred as partners and were transferred more money in a subsequent trust game. This interesting study implies that the benefits of behaving altruistically depends upon how much one is ready to contribute. The shared reward dilemma occurs in Jiménez et al. (2009), when the PDG is supplemented with a second stage in which a fixed reward is equally distributed among all cooperators.

Though previous (empirical and theoretical) studies verify that the mechanism of punishment can elevate public cooperation, the definition mathematical modeling of costly punishment and its resulted dilemma is still a socioeconomically relevant question. Its mechanism designing is still elusive since many undiscovered factors may be overlooked. Here relying on the framework of two-player PDGs, we define and investigate the role of shared punishment cost in influencing the emergence of cooperation from our point of view. Different with the shared cost in the cost-benefit game where a strategy determines how much cooperative effort an individual contributes (Brown and Vincent, 2008), our shared cost means that all the punishers equally share the cost of punishment, considering the fact that retaliation from one's opponent may be common in the real society. In the real social systems where the cost for the punishment may mean the retaliation from the punished defectors. When the defectors being punished have no idea about the punisher, they may make reprisals randomly. For the conditions where strategies (cooperation or defection) after the game round are public, the cooperators may be the victims of retaliation if only cooperators have the option to punish. Based on these situations, here we assume that only cooperators will share the cost for the punishment and defectors will be punished by cooperators. Thus in the two-strategy game of cooperators and defectors, it is clear that cooperators are the punishers and defectors will incur a loss of benefits due to the punishment. We aim to have a closer look at the nature of shared cost of punishment in the two-strategy situations described by the PDGs.

The rest of the paper proceeds as follows. Section 2 of our paper briefly describes the shared cost of punishment and outlines how the game was altered for our application. Theoretical results are contained in Sections 3 and 4, and a discussion of the conclusions is found in Section 5.

2. Model settings

As for the game model, we consider the two-player game contested by players who can make an option from two strategies, C (e.g. cooperation) and D (e.g. defection). In general, a C-player interacting with another C-player receives the benefits of R. If she interacts with a D-player, she obtains the payoff S. Similarly, the D-player receives T from the C-player and P from other D-players. The payoff gained by each player depends on the following payoff matrix,

	Cooperation	Defection
Cooperation	R	S
Defection	T	P

Parameters $T > R > S > P$ can perfectly describe the essence of the PDG, where defection is the best choice for players. For simplicity and without loss of generality, we adopt the parameter settings of R=1, $1 < T = b < 2$, and $P = S = 0$.

	Cooperation	Defection
Cooperation	1	0
Defection	b	0

In many situations, the number of players who participate in the evolutionary games is finite. So we consider a scenario as follows. From time to time, N players are chosen randomly from a sufficiently large and well-mixed population consist of n_c cooperators and n_d defectors, among which the fraction of cooperators is x_c ($0 \le x_c \le 1$).

We denote by P_c and P_d the expected payoffs for a cooperator (C) and defector (D) respectively. The payoffs for a cooperator and defector in the N-player group are as follows

$$\begin{cases} P_c = R(n_c - 1) + Sn_d \\ P_d = Tn_c + P(n_d - 1) \end{cases} \tag{1}$$

On the basis of the traditional PDGs, here we add some new parameters which can describe the shared cost for establishing punishment, the damage of suffering punishment, and the cost for establishing punishment. In many real circumstances, defectors usually receive a fixed penalty independent of the numbers of free-riders and contributors, for example, when free riding a subway. Here we employ the parameter β to define the fixed damage from suffering punishment. On the other side, the cooperators' righteous action toward free-riders is costly, and this altruistic punishment cost suffered by all cooperators is represented by parameter α. Combined with the settings of R=1, $1 < T = b < 2$, and $P = S = 0$, the payoffs of the two strategies can be calculated respectively by

$$\begin{cases} P_c = (n_c - 1) - \frac{\alpha}{n_c} \\ P_d = bn_c - \beta \end{cases} \tag{2}$$

According to the settings, defectors will suffer a punishment in the form of payoffs reduction β ($\beta \ge 0$). Here, n_c denotes the number of cooperators among the players. α/n_c is the payoff loss which is undertaken by each cooperator. By introducing these parameters of β and α, this payoff structure can describe the proposed idea where the punishment on defectors and share cost undertaken by cooperators coexist in an infinite population.

3. Replicator dynamics analysis

We now focus on the strategy evolution dynamics of the groups composed by these two types of players. Based on the replicator