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Predator–Prey–Subsidy Population Dynamics on Stepping-Stone Domains

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Abstract

Predator-prey-subsidy dynamics on stepping-stone domains are examined using a variety of network configurations. Our problem is motivated by the interactions between arctic foxes (predator) and lemmings (prey) in the presence of seal carrion (subsidy) provided by polar bears. We use the *n*-Patch Model, which considers space explicitly as a "Stepping Stone" system. We consider the role that the carrying capacity, predator migration rate, input subsidy rate, predator mortality rate, and proportion of predators surviving migration play in the predator-prey-subsidy population dynamics. We find that for certain types of networks, added mobility will help predator populations, allowing them to survive or coexist when they would otherwise go extinct if confined to one location, while in other situations (such as when sparsely distributed nodes in the network have few resources available) the added mobility will hurt the predator population. We also find that a combination of favorable conditions for the prey and subsidy can lead to the formation of limit cycles (boom and bust dynamic) from stable equilibrium states. These modifications to the dynamics vary depending on the specific network structure employed, highlighting the fact that network structure can strongly influence the predator-prey-subsidy dynamics in stepping-stone domains.

Keywords: predator-prey dynamics; allochthonous resource subsidy; population dynamics; non-equilibrium dynamics; network structure in ecology; stepping-stone model

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1. Introduction

The Arctic is home to a unique ecosystem with many terrestrial and marine animals, and plant-life. One species living in the Arctic is the arctic fox (*Alopex lagopus*), a predator. The arctic fox's diet consists of lemmings (*Cricetidae* family), and certain birds and bird eggs [26, 27]. They also consume seal (*Phocidae* family) carrion, a nutritional subsidy discarded by polar bears (*Ursus martimus*) [26, 23]. The arctic fox is a migratory animal, travelling from areas where lemmings breed to areas in which polar bears live. The arctic fox migrates every 3–4 years in September following random paths instead of a specific pattern [36] and migration usually finishes by mid-February. During its migration, there is a high death rate due to lack of food, trapping, and diseases such as rabies [36], yet during the winter, when migration does not occur, these are still factors in the death rate [24].

Single-species models relevant to laboratory studies in particular can reflect telescoping effects influencing population dynamics in the real world [22]. Verhulst proposed that a self-limiting process should operate when a population becomes too large [22, 33], and suggested logistic growth in a population. The resulting Logistic Equation was later derived from first principles by Lotka [3, 17], and was first used in a model to describe two species competing for the same resources [2]. Shortly after this, Lotka and Volterra derived the first model of predator-prey interactions, known as the Lotka-Volterra Model [22, 34], under the assumptions: (i) the prey without predation grows unboundedly in a Malthusian way; (ii) predation reduces the prey's per capita growth rate by a term proportional to the prey and predator populations; (iii) in the absence of any prey for sustenance the predator's death rate can be modeled as an exponential decay function; (iv) the prey's contribution to the predators' growth rate is proportional to the available prey as well as the population of the predator. This model unrealistically predicts that the prey will grow without bound in the absence of predators. For this reason, the inclusion of the logistic equation to prevent the prey from growing in an unbounded manner is often used in variants of the Lotka-Volterra equations.

One variation to the Lotka-Volterra model was suggested by Solomon [31], and a decade later by Holling [9, 10]. Since there is a limit to the amount of prey which can be consumed by a predator in a finite amount of time, they proposed that the predator equation should involve growth of a rational functional form [23, 3] involving Holling's Disk Equation [3, 1]. Such a system was the motivation for a predator-prey-subsidy model [23]

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \theta\left(\frac{x}{x+s+h}\right)y, \tag{1.1}$$

$$\frac{ds}{dt} = i - \gamma s - \psi\left(\frac{s}{x+s+h}\right)y, \tag{1.2}$$

$$\frac{dy}{dt} = \left(\frac{\epsilon\theta x + \eta\psi s}{x+s+h}\right)y - \delta y, \tag{1.3}$$

where $x(t)$, $y(t)$, $s(t)$ are the size of the prey, predator, and subsidy populations at each value of time. For a list of the parameters used, see Table 1. In (1.2), i is the rate at which the