Mathematical Biosciences 293 (2017) 21–28

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

Review

Time-dependent peristaltic analysis in a curved conduit: Application to chyme movement through intestine

V.K. Narla[a,*], K.M. Prasad[a], J.V. Ramana Murthy[b]

[a] Department of Mathematics, GITAM University, Hyderabad 502329, India
[b] Department of Mathematics, National Institute of Technology, Warangal, India

ARTICLE INFO

Article history:
Received 25 February 2017
Revised 24 July 2017
Accepted 15 August 2017

Keywords:
Peristalsis
Curved pumps
MHD
Reflux
Trapping
Pumping efficiency

ABSTRACT

A theoretical model of time-dependent peristaltic viscous fluid flow through a curved channel in the presence of an applied magnetic field is investigated. The results for stream function, pressure distribution and mechanical efficiency are obtained under the assumptions of long wavelength and low Reynolds number approximation. Pressure fluctuations due to an integral and a non-integral number of waves along the channel length are discussed under influence of channel curvature and magnetic parameter. Two inherent characteristics of peristaltic flow regimes (trapping and reflux) are discussed numerically. The mechanical efficiency of curved magnetohydrodynamic peristaltic pumping is also examined. The magnitude of pressure increases with an increasing channel curvature and magnetic parameter. Reflex phenomenon is analyzed in the Lagrangian frame of reference. It is observed that reflex in the curved channel is higher than in the straight channel. The trapped fluid in a curved channel is studied in the Eulerian frame of reference and it contains two asymmetric boluses. The size of the lower bolus grows and the upper bolus decreases with increasing effect of magnetic strength. Pumping efficiency of the peristaltic pump is low for curved channel flow than for straight channel flow. Also, the pumping efficiency is comparatively low at the high effect of the magnetic parameter.

1. Introduction

The fluid mechanics of peristaltic flow has been studied for many years by many researchers because of its practical importance in physiology and industry. Several mathematical and experimental models have been developed to understand the fluid mechanical aspects of peristaltic motion. The mathematical models obtained by a train of waves in an infinitely long two-dimensional symmetric and/or asymmetric ducts containing a Newtonian and/or non-Newtonian fluid have been investigated by many researchers. Shapiro et al. [1] studied peristaltic transport using wave frame of reference under long wavelength approximation. Takabatake et al. [2] investigated two-dimensional peristaltic flow in a channel using finite-difference technique employing the upwind successive-over-relaxation method. The boundary element method was successfully employed by Pozrikidis [3] to study Stokes flow model of peristalsis. Rathish et al. [4] developed a stream function-vorticity ($\psi - \omega$) formulation of peristaltic flow in a two-dimensional channel. Peristaltic motion in two immiscible layers of fluid has been studied for Newtonian model by Brasseur et al. [5], and for non-Newtonian models by Prasad et al. [6,7]. Li and Brasseur [8] presented non-steady peristaltic transport of food bolus of Newtonian type in the esophagus by considering finite length tube based on lubrication theory. Eytan et al. [9] developed a Stokes flow model in a two dimensional peristaltic channel to simulate intra-uterine fluid motion in a sagittal cross section of uterus. Tripathi et al. [10] studied unsteady peristaltic motion of MHD non-Newtonian fluid (Jeffrey model) in a finite length cylindrical tube. Some of studies undertaken by Tripathi and coworkers address peristaltic transport of different fractional viscoelastic models to simulate chem movement in intestine [11–13]. In recent studies, Bandopadhyay et al. [14] presented peristaltic analysis by considering applied electric field in microfluidic channels. Tripathi et al. [15] extended this model by considering external magnetic field. In another work Tripathi et al. [16] developed a mathematical model to study viscoelastic physiological fluids through capillary altered by electroosmosis.

The study of fluid mechanics of biological fluids in the presence of magnetic field (Biomagnetic Fluid Dynamics) is a major area of interest for researchers. Some of the applications of magnetic fields in physiology are magnetotherapy, magnetic resonance imaging, magnetic devices and targeted drug delivery systems in the treatment of cancer. In view of such wide ranging applications, several

* Corresponding author.
E-mail address: vknarla@gmail.com (V.K. Narla).

http://dx.doi.org/10.1016/j.mbs.2017.08.005
0025-5564/

authors have investigated magnetic field effects for different fluid models. The classical work of Haik et al. [17] initiated a mathematical formulation for bio-magnetic fluid flow by considering the fluids are electrically poor conductors and the flow is affected only by the magnetization of the fluid. The model was extended by Papadopoulos et al. [18] to reflect the curved square ducts of biomagnetic flow. Tzirtzilakis [19,20] improved this model by incorporating both magnetization and electrical conductivity of the biofluid. In the context of peristalsis, several authors have investigated an applied magnetic field effects for different fluid flow models to describe fluid flow undergoing peristalsis with prescribed wall motions [21–25]. There have been a couple of investigations of peristaltic flow in curved channels. Sato et al. [26] developed a flow model in a two-dimensional curved channel undergoing peristalsis when the wave length is sufficiently long compared with channel width and the inertial effect is negligibly small. Subsequent studies based on this framework has been extended for many problems including heat transfer analysis, non-Newtonian fluid, magnetic effect and nano-fluids [27–36]. Tripathi et al. [37] investigated both analytically and computationally the peristaltic transport of Nakamura-Sawada bi-viscosity non-Newtonian fluid in a curved tube. All these studies in curved channels of peristaltic transport of Newtonian/non-Newtonian fluids have analyzed the problem in Eulerian frame of reference by neglecting the local dynamics such as spatial-temporal variations. Ramanamurthy et al. [38] have examined unsteady peristaltic flow dynamics in curved channels.

The aim of the present work is to discuss the time-dependent flow dynamics of MHD fluid in a curved channel driven by peristalsis. This model is based on the long wave length and low Reynolds number flow approximation but the solution procedure is time-dependent where the time is introduced through the wall motility at the boundaries. The model is applicable to transport of chyme in the intestine, blood flow through aorta and a roller blood pump. The problem is first modeled and then solved analytically for stream function. Pressure variations has been plotted for integral and non-integral of waves along the channel walls. Trapping, reflex and pumping efficiency are discussed. The effects of magnetic and curvature parameter on the characteristics of the resulting flow field are analyzed.

2. Mathematical model

The incompressibility condition and momentum equation for MHD fluid are, respectively, written as,

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\frac{\partial \mathbf{V}}{\partial t} - \mathbf{V} \times (\nabla \times \mathbf{V}) + \frac{1}{2}\nabla(|\mathbf{V}|) = -\frac{1}{\rho}\nabla p - \nu(\nabla \times \nabla \times \mathbf{V}) + (\mathbf{J} \times \mathbf{B}). \tag{2}$$

In the above equations, $\mathbf{V}$ is the velocity vector, ρ is the fluid density, ν is the kinematic viscosity, $\mathbf{J}$ is the current density, $\mathbf{B}$ is the applied magnetic field. The cartesian co-ordinate system (X', Y', Z') is related to the toroidal co-ordinate system (x, r, z) by the relations

$$X' = (R + r)\cos(x/R), \; Y' = (R + r)\sin(x/R), \; Z' = z. \tag{3}$$

Consider a two-dimensional flow of an incompressible viscous fluid in a curved channel of width $2a$ coiled in a circle with center O and radius R. There is no component in z direction. An external magnetic field of strength $\mathbf{B}$ is applied in radial direction as shown in Fig. 1. The fluid-wall interface is time-dependent and is given as follows:

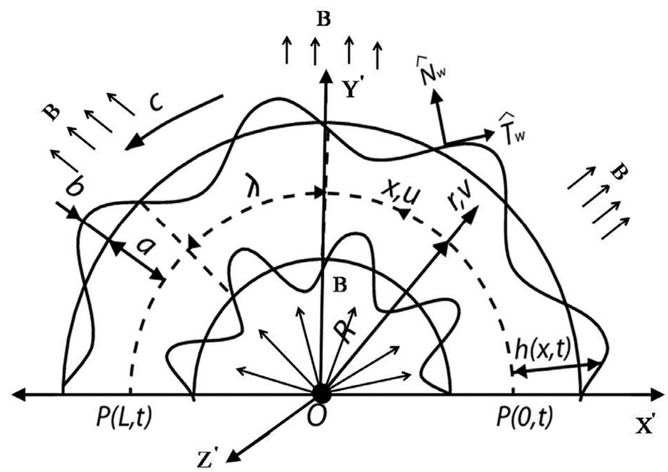

Fig. 1. Schematic view of peristaltic wave in curved channel.

$$r = \pm h(x, t) = \pm a \pm b\ cos\left[\frac{2\pi}{\lambda}\left(x - ct\right)\right]. \tag{4}$$

Here, x is the axial distance, a is the radius of the stationary curved channel, b is the wave amplitude, λ is the wave length, t is the time, c is the velocity of the wave, and h is the radial displacement of the wave from the centerline.

Neglecting the displacement currents, the Maxwell equations and the generalized Ohm law are

$$\nabla \cdot \mathbf{B} = 0, \; \nabla \times \mathbf{B} = \mu_m \mathbf{J}, \; \nabla \times \mathbf{E} = 0, \; \mathbf{J} = \sigma_e(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \tag{5}$$

where, μ_m is the magnetic permeability, $\mathbf{E}$ is the electric field and σ_e is the electric conductivity.

The following assumptions are applied:

- The quantities ρ, μ_m and σ_e are all constant throughout the flow.
- The applied magnetic field $\mathbf{B}$ is varying inversely with radial distance as $\mathbf{B}_r = \frac{B_0}{r+R}\widehat{e}$, in which B_0 is the magnitude of $\mathbf{B}$ and R is radius of the curved channel.
- The magnetic Reynolds number is sufficiently small, hence the induced magnetic field is negligible compared with the imposed magnetic field.
- The electrical field $\mathbf{E}$ is assumed to be zero.

Based on the above assumptions, the electromagnetic body force (Lorentz force) involved in Eq. (1) can be obtained as

$$\mathbf{J} \times \mathbf{B} = \sigma_e(\mathbf{V} \times \mathbf{B}) \times \mathbf{B} = \sigma_e[(\mathbf{V} \cdot \mathbf{B})\mathbf{B} - (\mathbf{B} \cdot \mathbf{B})\mathbf{V}] = -\sigma_e\left(\frac{B_0}{r + R}\right)^2 u\widehat{e_x}. \tag{6}$$

In view of the above Eqs. (1)-(6), the governing equations for MHD incompressible viscous fluid flow are given as:

$$R\frac{\partial u}{\partial x} + \frac{\partial}{\partial r}\{(r + R)v\} = 0, \tag{7}$$

$$\frac{\partial u}{\partial t} + \frac{Ru}{(r + R)}\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} + \frac{uv}{r + R} = -\frac{R}{\rho(r + R)}\frac{\partial p}{\partial x} + \nu\left[\left(\frac{R}{r + R}\right)^2\frac{\partial^2 u}{\partial x^2} + \frac{1}{r + R}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} - \frac{u}{(r + R)^2} + \frac{2R}{(r + R)^2}\frac{\partial v}{\partial x}\right] - \sigma_e\left(\frac{B_0}{r + R}\right)^2 u, \tag{8}$$

$$\frac{\partial v}{\partial t} + \frac{Ru}{(r + R)}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} - \frac{u^2}{r + R} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$