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On the spider that spits the solution of a nonsmooth oscillator

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ABSTRACT

Spitting spiders (*Scytodes sp.*) spit a mixture of silk and glue at their prey during attack. In this note, we show that a nonsmooth oscillator can be used as a biomechanical model to describe the zig-zag patterns produced by the spit of the spider *Scytodes thoracica*.

1. Introduction

Spitting spiders (*Scytodes sp.*) are so called because they have the habit of ejecting from their fangs a mixture of silk and glue onto their prey [1]. The fluid is produced in venom glands and ejected through a very small orifice located at the proximal end of the fang. It is however not yet clear if the spit from *Scytodes sp.* contains some venom or not. The envenomation of prey may indeed occur after its spit-immobilization. The spitting performance parameters of the species *Scytodes thoracica* have been measured bySuter and Stratton in [2]. The spider *S. thoracica* spits the fluid at speed V as high as 28 m/s (mean ± SE: 10.32 ± 1.99 m/s), the fang oscillates at a frequency ω up to 1700 Hz (mean ± SE: 826.3 ± 102.7 m/s) and the material is deposited during a time T less than 35 *ms* (mean ± SE: 25.2 ± 1.4 m/s). The right part of Fig. 1 is a composite that shows the spider *S. thoracica* and on the right side its spit produced as two zig-zag patterns, one from each fang. It shows also on the left side the enlarged spit pattern produced during one episode by one left chelicera/fang. The form of the patterns is a consequence of the spider's fangs' oscillations. More precisely, the lateral excursions of the expectorated silk are caused by the fang's oscillations and those oscillations are spread across the target space by the ventral-to-dorsal motion of the chelicera that bears the fang. Suter and Stratton have proposed a hydrodynamic forcing model that could explain this kind of material ejection. The fang is constrained by its hinge to one axis of rotation. Let α be the angle of the fang relative to its position $\alpha_0 = 0$ at rest. The biomechanical model proposed in [2] is:

$$m\alpha'' + c\alpha' + k\alpha + j\alpha^{2.4} = c(\alpha, \alpha') \tag{1}$$

with $m > 0$, $c > 0$, $k > 0$ and $j > 0$. The terms $m\alpha''$, $c\alpha'$, $k\alpha$ and $c(\alpha, \alpha')$ are used to represent respectively the change in angular momentum, the damping force, the tension due to elasticity and the hydrodynamic force arising from the forceful expulsion of fluid from the orifice of the venom duct. Moreover, the authors use the nonlinear function $j\alpha^{2.4}$ to represent the bookend forces that constrain the fang's motion at the extreme of its excursions. This nonlinear oscillator can indeed explain the high frequency of the oscillations because the size of a fang is particulary small for *Scytodes* species [3]. The zig-zag patterns suggest also a discontinuity in the angular speed α' at each time t at which a fang reaches the limits of its feasible motion. Such behavior cannot be obtained from the sinusoidal solution of a smooth oscillator described by a differential equation like in (1) and must be investigated using tools from the nonsmooth dynamical theory (see e.g. [4–12]). The nonsmoothness of the dynamics of fang motion has been recently confirmed by Suter and Stratton [13] by using high-speed video. The fang motion which has been studied in great temporal detail (10,000 frames per second) is not roughly sinusoidal but really nonsmooth (see figure 18.7 in [13]). In this paper, we use the nonsmooth approach to represent the behavior of the system at the extreme of its excursions and show that the solution of a nonsmooth oscillator is indeed comparable to the spit of the spider (see Fig. 1).

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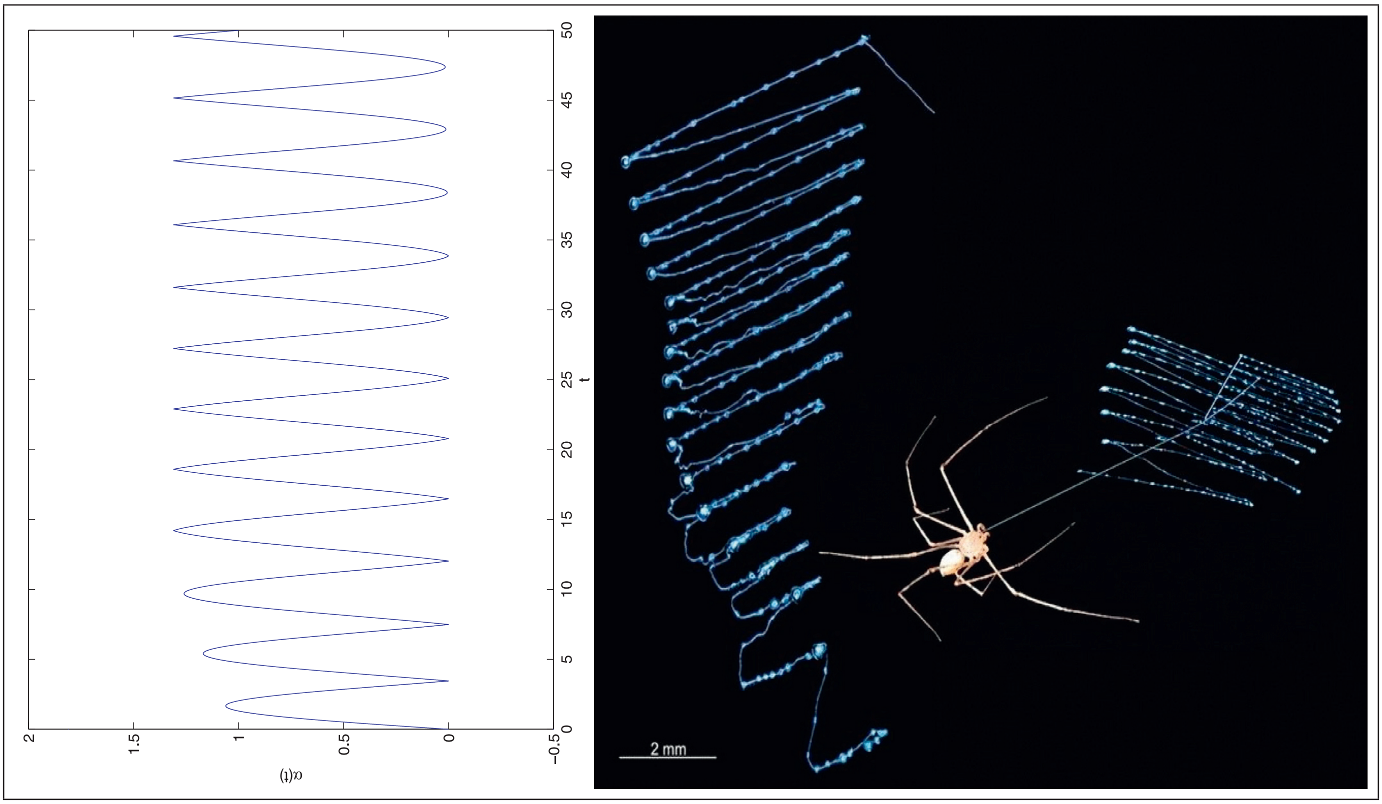

Fig. 1. Zig-zag oscillations of a nonsmooth oscillor (left) and zig-zag patterns (right) produced by the spit of *S. thoracica* (Source of the right photo - R. B. Suter and G. E. Stratton, Spitting performance parameters and their biomechanical implications in the spitting spider, *S. thoracica*, Journal of Insect Science: Vol. 9, Article 62, 1–15, 2009).

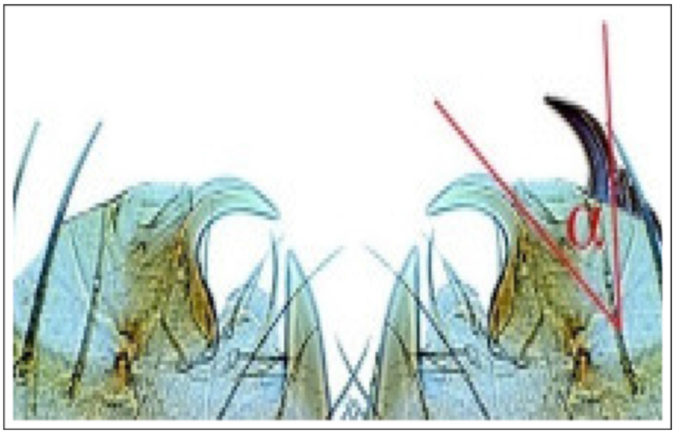

Fig. 2. Internal features: chelicerae and fangs (Source of the figure - R. B. Suter and G. E. Stratton, Spitting performance parameters and their biomechanical implications in the spitting spider, *S. thoracica*, Journal of Insect Science: Vol. 9, Article 62, 1–15, 2009).

2. Biomechanical model

Newton's second law for rotation requires that the net torque $\overrightarrow{\tau}$ of the system must be equal to the change in angular momentum $I\alpha''$ where I denotes the moment of inertia with respect to the axis of rotation $\overrightarrow{E}_3 = \overrightarrow{X}_3$ of the fang (see Figs. 2 and 3) and α'' is the angular acceleration:

$$I\alpha''(t)\overrightarrow{X}_3 = \overrightarrow{\tau}(t). \tag{2}$$

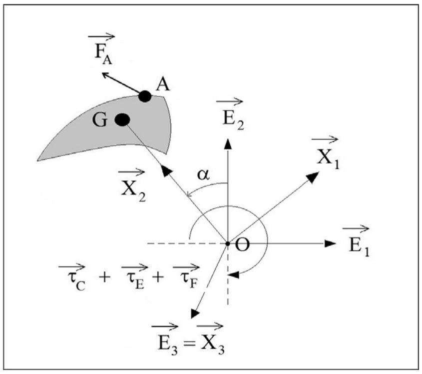

Fig. 3. Biomechanical model.

Recall that the parameter I is determined by the volume integral

$$I = \int_{\text{Fang}} ||\overrightarrow{x}||^2 \rho(\overrightarrow{x}) dV$$

where $\overrightarrow{x}$ is the position vector of a point X in the fang and $\rho(\overrightarrow{x})$ denotes the density of the fang at position $\overrightarrow{x}$. Note that the existence of the acceleration α'' cannot be expected everywhere since the angular velocity α' is discontinuous at each time t at which the fang reaches the limits of its feasible motion. The dynamics in (2) must thus be described here by a biomechanical model that treats possible velocity jumps on the same footing as smooth motions. The motion of the fang is indeed constrained along the axis of rotation by the mechanics of the fang's other attachments at the extremes of the excursions. We may thus consider a compact in-