Mathematical Biosciences 288 (2017) 71–83

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

# Mathematical modeling of an urban pigeon population subject to local management strategies

I. Haidar[a,b,d,*], I. Alvarez[b,c], A.C. Prévot[d]

[a] Sorbonne Universités, UPMC Univ Paris 06, Institut de Mathématiques de Jussieu, 4 place Jussieu, 75005, Paris, France
[b] Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6, F-75005, Paris, France
[c] Irstea, LISC, Aubière, France
[d] Centre d'Écologie et des Sciences de la Conservation (CESCO UMR 7204), Sorbonne Universités, MNHN, CNRS, UPMC, CP135, 57 rue Cuvier, 75005, Paris, France

ARTICLE INFO

*Article history:*
Received 4 May 2016
Revised 24 February 2017
Accepted 1 March 2017
Available online 6 March 2017

*Keywords:*
Biodiversity governance
Pigeons
Viability theory
Dynamical system

ABSTRACT

This paper addresses the issue of managing urban pigeon population using some possible actions that make it reach a density target with respect to socio-ecological constraints. A mathematical model describing the dynamic of this population is introduced. This model incorporates the effect of some regulatory actions on the dynamic of this population. We use mathematical viability theory, which provides a framework to study compatibility between dynamics and state constraints. The viability study shows when and how it is possible to regulate the pigeon population with respect to the constraints.



## 1. Introduction

Urban pigeon *Columba livia* populations can reach high densities in cities and cause cohabitation problems with urban citizens (see e.g. [21]). In response to social complaints, different regulation programs are implemented by local authorities to reduce perceived nuisance and help the coexistence between city dwellers and urban pigeons. These programs include different measures, from culling young or adult pigeons, to more welfare-based approaches (see e.g. [10]). One example is the building of public pigeon houses where food and nest-sites are provided for pigeons and where most laid eggs are removed or sterilized (see e.g. [20]). Complementarily, in order to control food resources for pigeons, pigeon feeding has been banned in most large cities. Nest and roost sites are also obstructed in some cities, which reduces resources for roosting and reproduction ([10]). However, the success of these regulation methods in controlling pigeons' numbers is not always achieved ([10]). In contrast, the so-called "pigeon problem" actually combines ecological and sociological issues (see e.g. [13]): human-pigeon coexistence is not solely a question of pigeon numbers (which could be dealt with by controlling), but could be considered, as proposed in [29], more successfully in terms of resilience and public perceptions of pigeons.

To be successful, any regulation strategy needs to anchor on a minimum knowledge about the pigeon ecology and the ecological consequences of the regulation. In fact, experimental evidence showed long-term side effects of some regulation methods implemented by local authorities (see e.g [11,20].). For example, egg removal may lead to an increase in laying frequency and in the total number of laid eggs in a year, together with an associated decrease in adult pigeon's body condition. Indeed, in [20], the authors explored the consequences of repeated egg removal on egg-laying cycles and egg quality of feral pigeons breeding in pigeon houses. During four years, they compared the egg quality and egg-laying cycles of pigeons breeding in several pigeon houses managed by egg removals to that of another population without egg removal. They observed that in pigeon houses with egg removal, the laying cycles were one-third the length of that in the pigeon houses without egg removal, leading to more laid eggs when reproduction is controlled. In addition, the genetic structure of pigeon populations indicates that pigeons can disperse from one site to another within a large metropolis (see e.g. [19]); this confirms empirical observations that reducing the number of pigeons in a particular site (whatever the method) is followed by the arrival of new individuals. Finally, as argued in [27], limiting breeding resources does not seem to guarantee limitation in pigeon number. The "pigeon problem" appears to be unsolvable by considering one single ecological variable at a time.

* Corresponding author.
*E-mail address:* Ihab.HAIDAR@imj-prg.fr (I. Haidar).

http://dx.doi.org/10.1016/j.mbs.2017.03.002

The purpose of this study is to propose a model to describe the evolution of a given fictive urban pigeon population impacted by regulation programs, by taking into account some of the different side effects highlighted in the literature (see above). In more detail, we model the dynamics of an urban pigeon population which is subject to two different regulation strategies: egg removal and resource limitation (food and/or nest-sites). We model urban citizen satisfaction through the tolerance of both a minimum and a maximum number of pigeons in a particular site. Here, we explore therefore how to maintain a given pigeon population under desirable constraints. Furthermore, we consider that the pigeon population is split into two sub-populations in two different sites. Because urban citizen satisfaction may differ from one site to another, we suppose that in each site the pigeon population is subject to different egg removal and resource limitation strategies. According to pigeon ecology, we also consider that pigeons disperse between these two sites, depending on both egg removal and resource limitation strategies adopted in each site. In fact, several studies exist on the patterns of pigeon dispersal confirming that dispersal is a natural mechanism in bird populations (see, e.g. [8,12,17,18]).

After introducing our model, we define our state constraint set which reflects urban citizen satisfaction. The viability theory of [4] offers an interesting insight in this context. It provides theoretical concepts and practical tools to study the evolution of dynamical systems under state constraint. The main purpose of viability theory is to find a “viability domain”, a subset of initial states from which there exists at least one evolution that remains in the state constraint set. Viability theory has been successfully used to model socio-ecological problem and study their governance, as in [6], [7] or [23] (see [5] for others references). We propose here to formulate the willingness of urban citizen about pigeon population in the mathematical viability framework. We then study when and how it is possible to propose satisfactory regulation. Using the viability algorithm developed in [9], we show the approximate viability kernel describing the possibility to control the density of an urban pigeon under social constraints.

The paper is organized as follows. Section 2 presents the model with a detailed description of all the parameters. Section 3 reformulates the management of the pigeon population as a viability problem and recalls some important definitions and theorems from the viability theory. Analytical and numerical results are given in Section 4. Section 5 discusses the ecological implications of the results of Section 4 in terms of the possibility of management of an urban pigeon population, under social constraints, along with some perspectives.

## 2. Model description

Before proceeding in the building of our model, we give a description about the pigeon life cycle. During the first year pigeon mortality is very high (see e.g [14,25].), notably in the first period (after fledging) when they are fed infrequently by their parents. Recruitment age of pigeons is not uniform among the pigeon population. It is important therefore, when modeling the dynamic of a pigeon population, to consider a structured model. Here, we consider that this population is divided into two classes (see Fig. 1): the juveniles and adults. Pigeons are considered as juveniles between fledging and recruitment. As soon as they first reproduce, they are considered as adults. We do not consider the intermittent reproduction. The mortality of juvenile pigeons is more important than that of adults. Let us denote by $x_j(t)$ and $x_a(t)$ the size of juvenile and adult pigeons, respectively, at time $t$. Their dynamics, in an infinitesimal time $dt$, is given by

$$x_j(t+dt) = x_j(t) + dt[nx_a(t) - m_j x_j(t) - px_j(t)]$$
$$x_a(t+dt) = x_a(t) + dt[-m_a x_a(t) + px_j(t)] \qquad (1)$$

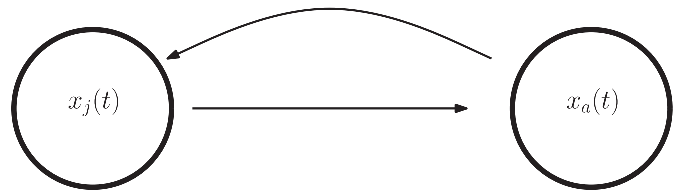


**Fig. 1.** Life cycle of a pigeon population structured in two classes.

where $n$ and $m_a$ denote the reproduction and mortality rates of adult pigeons and $m_j$ the mortality rate of juvenile pigeon. The parameter $p$ denotes the transfer rate from juvenile to adult class (i.e. recruitment rate).

As mentioned in the introduction, we assume that local authorities adopt both egg removal and resource limitation strategies to control the pigeon population. By the following, we list the principal impacts of these management strategies on the pigeon ecology:

- egg removal may lead to an increase in laying frequency and the total yearly number of laid eggs,
- egg removal may lead to a decrease in the body conditions and egg quality of adult pigeons, and therefore lower adult and juvenile survival,
- egg removal may impact the recruitment rate, through juvenile survival and numbers
- resource limitation may increase both adult and juvenile mortality rates, may decrease adult reproduction rate, and may decrease the recruitment rate

In order to incorporate these observations in the starting Eq. (1), we propose the following model

$$\dot{x}_j = n(x_a, r, s)x_a - m_j(x_j, r, s)x_j - p(x_j, r, s)x_j$$
$$\dot{x}_a = -m_a(x_a, r, s)x_a + p(x_j, r, s)x_j, \qquad (2)$$

where $r$ and $s$ denote, respectively, the egg removal and resource limitation strategies. Note that, in order to lighten the presentation of this model, we omit the time dependency of $x_a$, $x_j$, $r$ and $s$. Knowing that these control strategies may differ from one site to another, our model must incorporate a spatial distribution of the pigeon population. Here, we are limited to two sub-populations of urban pigeons, having different rates of reproduction and mortality. Furthermore, knowing that

- egg removal and resource limitation encourages the dispersal of pigeons,

and starting from Eq. (2), we propose the following model:

$$\dot{x}_{ji} = n_i(x_{ai}, r_i, s_i)x_{ai} - m_{ji}(x_{ji}, r_i, s_i)x_{ji} - p_i(x_{ji}, r_i, s_i)x_{ji} - \sum_{k=1}^{2}(-1)^{i+k}x_{jk}\phi_{jk}\left(x_{jk}, s_k\right)$$
$$\dot{x}_{ai} = -m_{ai}(x_{ai}, r_i, s_i)x_{ai} + p_i(x_{ji}, r_i, s_i)x_{ji} - \sum_{k=1}^{2}(-1)^{i+k}x_{ak}\phi_{ak}(x_{ak}, r_k, s_k) \qquad (3)$$

where $x_{ji}$ and $x_{ai}$ denote the size of juvenile and adult pigeon of population subject to removal strategy $r_i$ and resource limitation strategy $s_i$, for $i = 1, 2$. The function $n_i(\cdot)$ describes the reproduction of adult pigeon; $m_{ai}(\cdot)$ and $m_{ji}(\cdot)$ describe the mortality of adult and juvenile pigeon, respectively, for $i = 1, 2$. The function $p_i(\cdot)$ represent the transfer rate from juvenile to adult, for $i = 1, 2$. The functions $\phi_{jk}$ and $\phi_{ak}$ represents the dispersal rate of juvenile and adult pigeons from population $i$ to $k \neq i$, for $i, k \in \{1, 2\}$.