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A topological look at human trabecular bone tissue

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ABSTRACT

Bone quality is affected by trabecular architecture at microscopic level. Various abnormalities of bone tissue lead to altered strength and to an increased susceptibility to fracture, such as Osteoporosis and Osteoarthritis, two major health burdens of our society. These are two complex musculoskeletal diseases that mainly concern bone tissue. In the last twenty years, there has been a growing interest in finding an appropriate topological model for the micro-architecture of trabecular bone tissue. In particular, we prove that these models involve general topological spaces. The appropriate notion to deal with is that of CW-complex.

1. Introduction

One of the largest controversial issues of the scientific community is the assessment of comprehensive relations between bone density and bone quality (see, for instance [27]), which is affected by numerous factors [8]. Various abnormalities in the architecture of bone tissue lead to altered strength and to an increased susceptibility to fracture, such as Osteoporosis (OP) and Osteoarthritis (OA). The former is a metabolic condition characterized by loss of trabecular bone leading to an increased fragility and a propensity to fracture. By contrast, OA is characterized by progressive articular cartilage loss and osteophyte formation with subsequent remodelling and overgrowth of the adjacent bone. In spite of the major progress in the last few decades, we still have much to learn about these diseases. Although mechanical effects are claimed to be their main cause, we believe that a global approach to the geometry of trabecular micro-architecture might give significant information. Similarly to the effects of mechanical forces in trabecular bone remodelling [8], a fundamental, unexpected role might be given by Topology.

Human skeletal bone is one of the most common connective tissue of biological human structure. In relation to the internal microstructure, it can be divided into two main types: *compact* in the cortical zone and *trabecular* in the internal zone. Cortical bone will not be discussed in this article. Conversely, trabecular bone tissue is a hierarchical, spongy and porous material composed of hard and soft tissue components, which can be found in the vertebrae and the ends of the long bones, such as the femur, tibia and radius. The study of trabecular bone is important because age-related fractures primarily occur at trabecular bone sites, such as the proximal femur (hip), the distal radius (wrist), and vertebral bodies (spine). The matrix of trabecular bone is organized as a three-dimensional porous network of interconnected struts called *trabeculae*, given that tabecula means "little beam" in Latin.

The pores between trabeculae are filled with bone marrow. Because of its higher porosity, trabecular bone is somewhat weaker than "solid" cortical bone, but it is also much lighter. The bone in birds, for example, is primarily trabecular. The porosity of the trabecular network varies with anatomical location, biomechanical function, and age. The structure of trabecular bone and its mechanical behaviour is similar in many respects to that of engineering foams and other cellular materials, including wood. In the past, a particular type o polyhedron has been applied to bone, i.e., the truncated octahedron, an Archimedean solid. It is characterized by 14 faces, 8 regular hexagons and 6 squares, and can be generated by truncating the corners of a cube [33]. In 1887, Lord Kelvin [29] considered the question of how space could be partitioned into cells of equal volume with the least surface area between them (i.e., Plateau's soap-bubble problem) and discovered that the truncated octahedron would be the most suitable model [30]. In his honour, the truncated octahedron is also known as the *Kelvin cell*. Its structure can be used to model either the open-cell or the closed-cell structure. In the open-cell structure, the edges

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of the polyhedron are used, whereas in the closed-cell structure the faces are. Zysset et al. [35] investigated both the closed-cell and the open-cell versions of the Kelvin cell and discovered that the structures closely correspond to the mechanical behaviour of real trabecular bone for a wide range of volume fractions. Low-density trabecular bone resembles an open-cell and resemble a rod-like structure. High-density trabecular bone has a more plate-like structure with perforations through the plates. Thus, the Kelvin cell seems to be a suitable polyhedral model for the description of trabecular bone tissue. The orientation of the trabeculae can vary from almost equiaxed, e.g., in some sites in the proximal femur [10], to a nearly rectangular grid as in the vertebral body [14].

In some sense, the description of the trabecular structure in terms of the Kelvin cell is local. The regular truncated octahedron just gives a local picture of the architecture of the trabecular bone tissue. Joining some of these polyhedral result in more complicated geometric configurations, which provides a major flexibility in the global description of bone tissue. Characterizing these configurations is still complicated. At first, it seems feasible to introduce some topological indicators which yield interesting information about their shape up to continuous transformations. For instance, we can say when one of these configurations may be transformed to a sphere but not to a doughnut, which at the present stage is in fact relevant for applications. To distinguish two configurations from a topological points of view, it is possible to compute some invariants. If the invariant of two topological configurations is different, then the two configurations can not obtained one from the other via a continuous transformation. One of the most important of these topological invariants is the *Euler characteristic* or *Euler number*, which varies according to topological properties of our configurations. In order to investigate properties of trabecular architecture, where rod-like and plate-like structures are mixed, we apply the unifying notion of *CW-complex*, which generalizes the seminal topological approach initiated in [6,12] and [18]. This notion comes from Algebraic Topology and will be recalled in Section 3, as well as the computation of their Euler number.

As a consequence, all the results concerning the global behaviour of trabecular bone tissue - in the past twenty years - can be reinterpreted in terms of CW-complexes. Also, this notion provides a framework to understand the mechanics of natural structures, as well as novel scaffolds for engineering of highly porous tissues. For instance, in [15] the authors claim that the trabecular micro-architecture (predominantly plate-like) has a topological nature that allows transforming them into special CW-complexes that are called closed oriented genus g surfaces, where g is a non-negative integer number. Here, we will show that it is too restrictive to take into account only these CW-complexes.

In Section 2 we investigate the trabecular architecture in terms of the geometry of the Kelvin cells. In Section 3 we recall some basic facts and definitions from Algebraic Topology. In particular, we report some different ways of calculating the Euler characteristic of a CW-complex. In Section 4 we make some speculation on the Euler characteristic of sections of three-dimensional portions of bone tissue that the tissue might have. By the cellular structure, the sections can be deformed to the union of bunches of circles and isolated points. Such a configuration can be highlighted from images acquired with a high-resolution micro-tomographic imaging system. The number of bunches of circles and the Euler characteristic of the section yield the number of isolated points. In Section 4 we also prove that sections of compact, connected, oriented genus g surfaces do not have Euler characteristic greater than or equal to 2. In Section 2 we describe our methods. There we propose a linear interpolation of a rod-like and a plate-like structure in order to compute the normalized Euler number per unit total volume. This interpolation yields results that fit to the values measured via a 3D image analysis by micro-CT. We end our paper with some discussions in Section 5.

2. Methods and materials

In this section we report some experiments made on different bone specimens.

Eleven trabecular bone specimens were extracted from femoral heads of two patients subject to a hip arthroplasty surgery. One of them was obtained from patients with osteoporotic hip fracture (OP) and ten with moderate hip osteoarthritis (OA). Specimens were preserved in freezer at 10 °C for about a month. From the middle of the femur capita of each specimen, a slice of about 10 mm of thickness corresponding to the frontal plane was obtained and was kept 10 h in freezer. Subsequently, cubic specimens of 10 × 10 × 10 mm were obtained; they coincided with the principal stress trajectories in the loaded femur, according to Wolff's law. We recall that this law states that i) trabecular bone tends to be formed during growth and development in orientations that correspond to principal mechanical stresses acting on the bone, and ii) hypothetical mathematical laws can explain this process [32]. This specific protocol has been used extensively for different research projects: see [1,19–22].

All specimens were acquired with a high-resolution micro-tomographic imaging system (Skyscan 1072®-Zurich, Switzerland), referred to as desktop μCT-Scanner. Detailed instrumentation of the μCT can be found elsewhere [26]. The protocol employed X-ray tube settings of 100 kV and 98 μA with a nominal isotropic voxel resolution in the range of 11.24 μm - 14.66 μm and 400 projections were acquired over an angular range of 180°.

A 1-mm aluminium filter and beam-hardening correction algorithm were used to compensate for artifacts associated with the use of a X-ray source. Serial 8-bit 1024 × 1024 pixel image sequences were reconstructed using a cone-beam algorithm based on Feldkamp algorithm (SkyScan Cone Recon ®software). Data were collected using SkyScan proprietary software CTan®.

The grey-value images were binarised by using a newly developed technique of thresholding, a code in LabView (Austin, Texas-USA) was implemented [22]. A second segmentation was performed by NI Vision (Austin, Texas-USA) to separate bone from background using a locally adaptive thresholding procedure. A background correction algorithm was applied to keep only the largest connected bone-component and to remove small particles arising from noise and artefacts. Finally, by using SkyScan proprietary Software CT-analyser, allowed to obtain histomorphometric parameters of the specimens, such as the Trabecular Number (Trab.N).

Motivated by inspection of scanning electron microscopic images of trabecular microstructures and by the fact that the morphologic and elastic symmetry is close to orthotropy [34], a tetrakaidecahedral geometry [11] was selected to model the trabecular architecture. A 3D idealized microstructural model of trabecular bone [35] was employed therein. These cells filled in the 3D space and were connected by either all beams (rod-like trabecular bone model) or all plates (plate-like trabecular bone model): see Fig. 2.

As a practical simplification, without loss of generality for the model, we examined the cases where each cell had an equal edge length, l, and $\theta = 45°$. The choice of $\theta = 45°$, for simplicity, resulted in an isotropic microstructure of trabecular bone. To account for various volume fractions, the microstructural model consists of an open cell where beams occupy the edges and a closed cell where plates occupy the hexagonal faces of the cell. Volume fraction was calculated by using beams with circular cross-section, plates of uniform thickness and neglecting high order effects at the corners or edges.