

# Mathematical model for path selection by ants between nest and food source



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## ABSTRACT

Several models have been proposed to describe the behavior of ants when moving from nest to food sources. Most of these studies were based on numerical simulations with no mathematical justification. In this paper, we propose a mechanism for the formation of paths of minimal length between two points by a collection of individuals undergoing reinforced random walks taking into account not only the lengths of the paths but also the angles (connected to the preference of ants to move along straight lines). Our model involves reinforcement (pheromone accumulation), persistence (tendency to preferably follow straight directions in absence of any external effect) and takes into account the bifurcation angles of each edge (represented by a probability of willingness of choosing the path with the smallest angle). We describe analytically the results for 2 ants and different path lengths and numerical simulations for several ants.

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## 1. Introduction

One of the most interesting problems in mathematical biology is the study of the mechanisms employed by the multiple parts composing a biological system to create a collective self-organized behavior (see the books [3,6] and [30]). Not only different groups of animals such as ant colonies [13,18,32] or fish schools [22,23] exhibit this coordinated behavior but also human crowds [16,17]. Due to the complexity underlying such collective systems, it is necessary to understand well and to identify the key factors of the mechanism. For this reason the development of theoretical methodologies to study complex biological systems is a very active area of research and a rich source of new and interesting mathematical problems. In the last years, many studies about animal transportation networks have been performed focusing on different aspects such as topological properties of networks, network morphogenesis and growth, and behavior of network's users (for a complete explanation see the review article [26]).

For the particular case of ant colonies, several problems related to the formation of trail networks created by multiple species of ants have been studied [1,2,8,10,19]. Their patterns that can extend

up to a hundred meters from the nest result from the collective activity of thousands of individual workers laying and following pheromone trails as they forage their environment. Some previous studies show the efficient exploitation of resources by these limited individuals [4,8,10,12,26,34]. Although some species seem to have developed theoretically optimal tree-like networks, in some cases cycles are observed in the rooted foraging networks increasing the robustness of the network [7,20]. Our goal is to understand how can a group of cognitively limited individuals, with access to partial and local information only, build such efficient, large scale structures. From a mathematical point of view, we want to find out the minimal set of rules that a model must incorporate to give rise to networks similar to those of ant colonies. This question can be extended to other self-organized network systems, such as the galleries of ant nests [27], but also trail systems created by humans [15,17].

Several experimental and theoretical studies have been performed in order to solve this question. It has been investigated the individual and collective behavior of Argentine ants (*Linepithema humile*) when navigating graphs [13,25,33,34] and also the behavior of *Lasius niger* ants when bifurcation asymmetries are present [14]. In [31] several experiments were conducted with three different ant species to explore their responses to varying concentrations of pheromone in order to understand the importance of this parameter in the mathematical models. All

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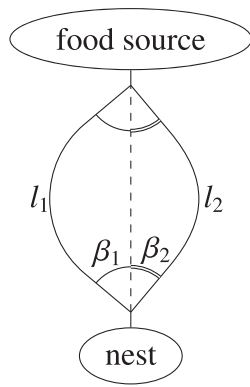


Fig. 1. A simple graph with two nodes.

these studies suggest that a mechanism for the formation of ant trails should include an attractive, durable signal (pheromone) as well as motion persistence, that is, the tendency to move straight in the absence of external information. These two factors have consequences for colony-level path selection, allowing ant colonies to efficiently choose shorter paths to food sources [11,33]. This kind of motion is known in the mathematical literature as *reinforced random walk* [9,21,24] and it is sufficient to reproduce the individual and collective movement patterns of ants on graphs [33]. The role of persistence has been stressed in other biological contexts, such as the motion of cells [28,29].

In a recent study, Vela-Pérez et al. [33] investigated the models of shortest path selection by ants that travel between two points: the nest and the food source. In particular, two simple graphs were considered: with two and with three nodes, being the last case equivalent to a two node network with a long path of length two. In the simplest two-node graph (see Fig. 1) it was considered that the length of both paths is the same (i.e.  $l_1 = l_2$ ). Moreover, it was assumed that the probability of the path selection only depends on the pheromone concentration. However, when making a decision, ants have to decide between two or more conflicting information sources or environmental effects and hence several options are open to them. Our goal is to investigate the influence of different path lengths and different angles in a bifurcation point, which extends the analysis performed in [33]. As it was proved experimentally for *Lasius niger* ants [14], ants preferentially follow the branch deviating least from straight and this effect increases as asymmetry increases in bifurcation points. Therefore, we consider an additional factor of path selection connected with angles  $\beta_1$  and  $\beta_2$  in order to study this mathematically.

The paper is organized as follows: in Section 2 we present analytical results related to the asymptotic dynamics in the case of two node networks. In Section 3, we describe the behavior for early time dynamics for the case of two node networks with edges of different lengths and different bifurcation angles with two ants. In Section 4, we present some numerical results for several ants in two node networks with edges of different lengths and different bifurcation angles. In Section 5, we summarize the final conclusions.

## 2. Mathematical analysis of two nodes network with one ant

Firstly, we investigate the asymptotic distribution of pheromone on the edges. Without loss of generality we may assume that  $l_2 = 1$  and  $l_1 = L$ . We also make the following assumptions:

- (1) ants move with a constant speed;
- (2) only one ant travels between nest and food source; that is the ant has to return to the nest before the next ant departs;

- (3) ants deposit pheromone uniformly;
- (4) the path selection is done in terms of pheromone concentration and the angle of the path;
- (5) the graph is symmetric, that is the angles of a given path near the nest and the food source are the same.

The amount of the pheromone on each graph's edge depends on the number of ants that followed the edge and the length of the path. We denote the amount of pheromone on an edge  $j = 1, 2$  at time  $t$  by  $\omega_j(t)$ . Following the idea presented in [33], we assume that the probabilities  $p_j$ ,  $j = 1, 2$  of choosing edge  $j$  are the following:

$$p_1(t) = \frac{q(k + \frac{\omega_1(t)}{L})^\alpha}{q(k + \frac{\omega_1(t)}{L})^\alpha + (1 - q)(k + \omega_2(t))^\alpha}, \tag{2.1a}$$

$$p_2(t) = \frac{(1 - q)(k + \omega_2(t))^\alpha}{q(k + \frac{\omega_1(t)}{L})^\alpha + (1 - q)(k + \omega_2(t))^\alpha}, \tag{2.1b}$$

where  $k$  is a pheromone detection threshold and  $\alpha$  is a non-linearity coefficient. A high value of  $\alpha$  means that even if one branch has only slightly more pheromone than the other, the ant will have a larger probability of choosing it. If  $\alpha=1$  the ants react in a linear, proportional way to pheromone concentration. The parameter  $k$  acts as a threshold for response to pheromone. For larger values of  $k$ , more marking is necessary for the choice to become significantly non-random (see [5–13]). The value of  $\omega_1(t)$  (respectively  $\omega_2(t)$ ) is increased in one unit each time the ant moves along the correspondent edge, representing the deposit of pheromone by the ant. Ants move one step at each time interval  $\Delta t$  that can be taken, without loss of generality, as  $\Delta t = 1$ .

The probability  $q$  reflects the willingness of choosing the first path because of the angle. Thus,  $q$  is a function of  $\beta_1$  and  $\beta_2$ . However, the precise formula of the function is neither known nor needed, and  $q$  can be treated as a parameter. It is reasonable to assume that  $q(\beta_1, \beta_2) = 1/2$ ,  $q > 1/2$  for  $\beta_1 > \beta_2$  and  $q < 1/2$  for  $\beta_1 < \beta_2$ . We also assume that the probability  $q$  is the same at the bifurcation points near the nest and near the food source.

### 2.1. Long time behavior

In order to simplify the arguments we assume that ants depart from the nest every time unit. This unit has to be taken large enough in order to allow the ant to reach food source and return to the nest by any edge. This assumption is not very important since we neglect pheromone evaporation. On the other hand, we will show later that the assumption that only one ant is traveling through the graph is extremely important.

It is easy to see, that

$$\begin{aligned} \omega_1(t + 1) - \omega_1(t) &= Lp_1, \\ \omega_2(t + 1) - \omega_2(t) &= p_2. \end{aligned} \tag{2.2}$$

Clearly the concentrations of pheromone on each edge are stochastic processes. Thus, summing over all possible realizations of the stochastic processes we calculate the expectations values of  $\omega_1$  and  $\omega_2$  and denoting them by  $\langle \omega_1 \rangle(t)$  and  $\langle \omega_2 \rangle(t)$ , respectively, we obtain from (2.2) the equations

$$\begin{aligned} \langle \omega_1 \rangle(t + 1) - \langle \omega_1 \rangle(t) &= L\langle p_1 \rangle, \\ \langle \omega_2 \rangle(t + 1) - \langle \omega_2 \rangle(t) &= \langle p_2 \rangle, \end{aligned} \tag{2.3}$$

where  $\langle x \rangle$  denotes the expectation of  $x$ .

We are interested here in the distribution of pheromone after a long time. If the amount of pheromone deposited by a single ant is of order  $O(1)$  then after  $N$  time units the value of  $\omega_j$ ,  $j = 1, 2$

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