Mathematical Biosciences 289 (2017) 130–141

Contents lists available at ScienceDirect

Mathematical Biosciences

journal homepage: www.elsevier.com/locate/mbs

Dynamics analysis of a delayed reaction-diffusion predator-prey system with non-continuous threshold harvesting

Xuebing Zhang[a,b], Hongyong Zhao[a,*]

[a] Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, People's Republic of China
[b] Department of Basic Courses, Huaian Vocational College of Information Technology, Huaian 223003, People's Republic of China

ARTICLE INFO

Article history:
Received 18 December 2016
Revised 9 April 2017
Accepted 17 May 2017
Available online 19 May 2017

Keywords:
Hopf bifurcation
Predator-prey system
Non-continuous harvesting
Stability
Reaction-diffusion
Delay

ABSTRACT

This paper deals with a delayed reaction-diffusion predator–prey model with non-continuous threshold harvesting. Sufficient conditions for the local stability of the regular equilibrium, the existence of Hopf bifurcation and Turing bifurcation are obtained by analyzing the associated characteristic equation. By utilizing upper-lower solution method and Lyapunov functions the globally asymptotically stability of a unique regular equilibrium and asymptotically stability of a unique pseudoequilibrium are studied respectively. Further, the boundary node bifurcations are studied. Finally, numerical simulation results are presented to validate the theoretical analysis.

1. Introduction

Harvesting of species is an important issue from ecological and economic perspectives [1–3]. It is commonly carried out in wildlife, fishery, and forestry management and in integrated pest management (IPM) programs. There is a wide range of interest in the use of bioeconomic models to gain insight into the scientific management of renewable resources such as fisheries and forests. Presently, there are numerous works on predator–prey systems with harvest terms [4–7].

Most models with harvesting consider either constant or linear harvesting functions and harvesting starts at $t = 0$. We remark that it is not very realistic to assume that harvesting starts at $t = 0$. In this regard, threshold policy harvesting in which harvesting starts only when a population has reached a certain threshold value T has been proposed (see [8–13]). It is believed that such harvesting function is more sound from the biological view point. Classically, such a harvesting function is defined as

$$\psi(x) = \begin{cases} 0, & x < T, \\ qx, & x > T. \end{cases} \tag{1.1}$$

In [8], Meza et al. studied the following Rosenzweig–MacArthur model with threshold harvesting

$$\begin{cases} \frac{dx}{dt} = rx(1 - \frac{x}{K}) - \frac{xy}{A+x}, \\ \frac{dy}{dt} = \frac{sA(x-J)}{(J+A)(x+A)} - \epsilon y, \\ x(0) = x_0 > 0, y(0) = y_0 > 0, \end{cases} \tag{1.2}$$

where

$$\epsilon = \begin{cases} 0, & x < T, \\ 1, & x > T. \end{cases} \tag{1.3}$$

The authors used a Liapunov functional approach to prove that the point in the sliding region is globally asymptotically stable under some conditions.

In [10], Zhang and Tang studied a Filippov ratio-dependent prey-predator model with an economic threshold. The sliding mode domain, sliding mode dynamics, and local sliding bifurcations including regular/virtual equilibrium bifurcations and boundary node bifurcations are studied. In [12], threshold policy is proposed to control pests. Research results show that as the threshold value varies, local sliding bifurcations including boundary node (saddle), tangency, and pseudo-saddle–node bifurcations occur sequentially, and global sliding bifurcations including buckling bifurcations of the sliding cycles, sliding crossing bifurcations, and pseudohomoclinic bifurcations can be presented.

The models mentioned above are mainly described by ordinary differential equations (ODEs). It is well known that delay differen-

* Corresponding author.
E-mail addresses: zxb1030@163.com (X. Zhang), hyzhao1967@126.com (H. Zhao).

http://dx.doi.org/10.1016/j.mbs.2017.05.007
0025-5564/

tial equations exhibit much more complicated dynamics than ODEs since it can cause the loss of stability and induce various oscillations, periodic solutions and chaos phenomena [14–21]. Delay due to gestation is a common example, because consumption of prey by a predator throughout its history generally governs the present birth rate of the predator. Therefore, more realistic models of population interactions should take into account the effect of delay. On the other hand, in real life, a species is spatially heterogeneous. This spatial dispersal is mainly due to resource limitation: in regions of high population density, food will become scarce and individuals will tend to migrate to regions of lower population density [22].

Motivated by the above discussions, following the works of authors in [23–27], we consider the following Leslie predator–prey system

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + ru(1-\frac{u}{K}) \\ \quad -\frac{muv(t-\tau_1)}{a+u^2} - \psi(u), & (x,t) \in \Omega \times (0,+\infty), \\ \frac{\partial v}{\partial t} = d_2 \Delta v + sv(1-\frac{v}{hu(t-\tau_2)}), & (x,t) \in \Omega \times (0,+\infty), \\ u(x,t) = \phi_1(x,t), v(x,t) \\ \quad = \phi_2(x,t), & (x,t) \in \Omega \times [-\tau, 0], \\ \frac{\partial u(t,x)}{\partial n} = \frac{\partial v(t,x)}{\partial n} = 0, & t > 0, x \in \partial\Omega \end{cases} \tag{1.4}$$

where u, v represent the prey and predator biomass, respectively. K is carrying capacity, r is intrinsic growth rate of prey, m is the capture rate of the predator, s is intrinsic growth rate of predator, a measures the extent to which environment provides protection to prey u, the parameter h is a measure of the food quality of the prey for conversion into predator births. τ_1 represents that the predator species need time τ_1 to possess the ability of predation after it was born, while τ_2 denotes the time taken for digestion of the prey, Δ denotes the Laplacian operator, Ω is a bounded domain, n is the normal vector that goes out of bounded domain Ω. The homogeneous Neumann boundary conditions indicate that there is no population flux across the boundaries. For the initial conditions, we assume that

$$\phi_j(s,x) \in \mathcal{C} = C([-\tau,0], X)$$

and X is defined by

$$X = \{u \in W^{2,2}(\Omega) : \frac{\partial u(t,x)}{\partial n} = \frac{\partial v(t,x)}{\partial n} = 0, x \in \partial\Omega\}$$

with the inner product $\langle \cdot, \cdot \rangle$. $\psi(u)$ is a harvesting term and defined as follows

$$\psi(u) = \begin{cases} 0, & u < T, \\ qu, & u > T, \end{cases} \tag{1.5}$$

where $q < r$ is harvest coefficient. It works as follows: when spatially population density of prey u is above a certain level or threshold T, harvesting occurs; when spatially population density of prey u falls below that level, harvesting stops.

By letting

$$\begin{aligned} &\bar{t} = rt, \bar{u} = \frac{u}{K}, \bar{v} = \frac{mv}{rK^2}, \bar{a} = \frac{a}{K^2}, \bar{s} = \frac{s}{r}, \bar{h} = \frac{mh}{rK}, \bar{\tau}_1 = \tau_1 r, \\ &\bar{\tau}_2 = \tau_2 r, \bar{d}_1 = \frac{d_1}{r}\Delta^2, \bar{d}_2 = \frac{d_2}{r}\Delta^2, \bar{x} = \frac{x}{\Delta}, \bar{T} = \frac{T}{K}, \bar{q} = qr \end{aligned} \tag{1.6}$$

and dropping the bars for the sake of simplicity, system (1.4) becomes the following system

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 \Delta u + u(1-u) - \frac{uv(t-\tau_1)}{a+u^2} - \psi(u), \\ \frac{\partial v}{\partial t} = d_2 \Delta v + sv(1-\frac{v}{hu(t-\tau_2)}), \end{cases} \tag{1.7}$$

To the best of our knowledge, few researchers have studied delayed reaction–diffusion with non-continuous threshold harvesting. In this paper, we will study the existence of equilibrium, stability and Hopf bifurcation of the equilibrium.

The paper is organized as follows. In Section 2, we give some preliminaries. In Section 3, we consider the existence of equilibrium points of system (1.7). In Section 4, we analyze the corresponding characteristic equations and investigate the local stability and the existence of Hopf bifurcation at a regular equilibrium. In Section 5, we use upper-lower solution method to derive sufficient conditions for the global asymptotic stability of the regular equilibrium of system (1.7). In Section 6, we discuss the global asymptotic stability of the pseudoequilibrium. In Section 7, we study sliding bifurcation. In Sections 8 and 9, we carry out numerical simulations to illustrate the main theoretical results. Section 10 concludes with a brief discussion.

2. Preliminaries

In [28], the authors have proposed some useful properties and definitions for Filippov systems which are described by ordinary differential equations. Here, we introduce these properties and definitions into partial differential equations, so that we can investigate system (1.7) in more details.

Letting $H(Z) = u - T$ with vector $Z = (u, v)^T$, and

$$\begin{aligned} F_{G^1}(Z) &= \left(d_1 \Delta u + u(1-u) - \frac{uv(t-\tau_1)}{a+u^2}, d_2 \Delta v \right. \\ &\quad \left. + sv\left(1 - \frac{v}{hu(t-\tau_2)}\right)\right)^T, \\ F_{G^2}(Z) &= \left(d_2 \Delta u + u(1-u) - \frac{uv(t-\tau_1)}{a+u^2} - qu, d_2 \Delta v \right. \\ &\quad \left. + sv\left(1 - \frac{v}{hu(t-\tau_2)}\right)\right)^T. \end{aligned}$$

Then system (1.7) can be rewritten as the following Filippov system

$$\frac{\partial Z}{\partial t} = \begin{cases} F_{G^1}(Z), & Z \in G_1, \\ F_{G^2}(Z), & Z \in G_2, \end{cases} \tag{2.1}$$

where $G_1 = \{Z | H(Z) < 0\}$, $G_2 = \{Z | H(Z) > 0\}$. Furthermore, the discontinuity boundary (or manifold) Σ_s separating the two regions G_1 and G_2 is described as $\Sigma_s = \{Z | H(Z) = 0\}$. The main characteristic of a Filippov system is that control is suppressed when the value of the threshold function is below a previously chosen threshold policy; above the threshold, control is applied, while in classical systems, control has been implemented all along. In the following, we call Filippov system (2.1) defined in region G_1 system S_1 and that defined in region G_2 system S_2.

The following definitions of all types of equilibria of Filippov system are necessary throughout the paper [29–32].

Definition 2.1. A point Z^* is called a regular equilibrium of system (2.1) if $F_{G^1}(Z^*) = 0$, $H(Z^*) < 0$ or $F_{G^2}(Z^*) = 0$, $H(Z^*) > 0$. A point Z^* is called a virtual equilibrium of system (2.1) if $F_{G^1}(Z^*) = 0$, $H(Z^*) > 0$ or $F_{G^2}(Z^*) = 0$, $H(Z^*) < 0$.

Definition 2.2. A point Z^* is called a pseudoequilibrium if it is an equilibrium of the sliding mode of system (2.1), that is, $(1-\lambda)F_{G_1}(Z^*) + \lambda F_{G_2}(Z^*) = 0$, $H(Z^*) = 0$, and $0 < \lambda < 1$, where

$$\lambda = \frac{\langle H_Z(Z), F_{G_1}(Z) \rangle}{\langle H_Z(Z), F_{G_1}(Z) - F_{G_2}(Z) \rangle}. \tag{2.2}$$

Definition 2.3. A point Z^* is called a tangent equilibrium of system (2.1) if $Z^* \in \Sigma_s$ and $\langle H_Z(Z^*), F_{G_1}(Z^*) \rangle = 0$ or $\langle H_Z(Z^*), F_{G_2}(Z^*) \rangle = 0$.