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# Consistency and bicharacteristic analysis of integral porosity shallow water models. Explaining model oversensitivity to mesh design

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ABSTRACT

The Integral Porosity and Dual Integral Porosity two-dimensional shallow water models have been proposed recently as efficient upscaled models for urban floods. Very little is known so far about their consistency and wave propagation properties. Simple numerical experiments show that both models are unusually sensitive to the computational grid. In the present paper, a two-dimensional consistency and characteristic analysis is carried out for these two models. The following results are obtained: (i) the models are almost insensitive to grid design when the porosity is isotropic, (ii) anisotropic porosity fields induce an artificial polarization of the mass/momentum fluxes along preferential directions when triangular meshes are used and (iii) extra first-order derivatives appear in the governing equations when regular, quadrangular cells are used. The hyperbolic system is thus mesh-dependent, and with it the wave propagation properties of the model solutions. Criteria are derived to make the solution less mesh-dependent, but it is not certain that these criteria can be satisfied at all computational points when real-world situations are dealt with.



## 1. Introduction

Two-dimensional porosity-based shallow water models for urban flood modelling have gained popularity over the past decade. With computational times reduced by two to three orders of magnitude compared to refined shallow water models, they appear as a promising option for upscaling the shallow water equations in the urban environment. Originally, these models incorporated only one type of porosity and were formulated in differential form (Defina, 2000; Guinot and Soares-Frazão, 2006; Hervouët et al., 2000). Most developments so far have focused on this isotropic, Single Porosity (SP) version (Benkhaldoun et al., 2016; Cea and Vazquez-Cendon, 2010; Finaud-Guyot et al., 2010; Viero and Mohammad Valipour, 2017). The methods proposed to address the anisotropy of the urban medium use several types of porosity instead of a single one. Such models include the Multiple Porosity (MP) model (Guinot, 2012) and the Integral Porosity (IP) model (Sanders et al., 2008). The salient features of the IP approach are that (i) a differential formulation for such models is deemed meaningless in that the urban medium is not continuous on the scale at which the porosity model is used, (ii) two types of porosity are distinguished: a storage porosity, that represents the volume fraction available for mass and momentum storage, and a connectivity porosity, that accounts for the connectivity of the urban medium, thus acting on the computation of fluxes. This formulation is well-suited to finite volume, shock-capturing numerical techniques. The latest developments available from the literature include depth-variable IP models (Özgen et al., 2016a) and the Dual Integral Porosity (DIP) model (Guinot et al., 2017). Laboratory and numerical experiments have shown the superiority of the IP approach over the SP (Kim et al., 2015). The DIP model yields improved wave propagation properties over the IP model (Guinot et al., 2017).

The IP/DIP approach allows the anisotropy of the urban medium to be characterized very easily via the connectivity porosity. In finite volume discretizations (that are the only family of discretizations proposed so far for such models), the connectivity porosity is defined for each cell interface from the intersection with building contours (Sanders et al., 2008; Schubert and Sanders, 2012). This makes its numerical value strongly dependent on the mesh design, as opposed to the SP (Guinot and Soares-Frazão, 2006) and MP (Guinot, 2012) approaches, that use a domain-based statistical definition for the porosity. In Sanders et al. (2008), various meshing strategies are proposed, all leading to different values for the connectivity porosities. While these strategies are compared in terms of computational effort, little is known on their influence on the accuracy of the porosity model apart from the study reported in Schubert and Sanders (2012)

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Although the differential form of the porosity equations is deemed meaningless in the integral approach, Guinot (2012); Guinot et al. (2017) show that the differential expression of the governing equations gives useful and accurate information of the wave propagation properties of the porosity model. However, only the one-dimensional version of the IP/DIP equations has been analysed (Guinot and Delenne, 2014; Guinot et al., 2017). No full 2D analysis has been provided so far, although the first steps of such an analysis were made in Lhomme (2006) for a particular case of the IP model. The purpose of the present paper is to provide such an analysis for both the IP and DIP model and to draw consequences in terms of IP/DIP solution behaviour and accuracy.

The need for a two-dimensional analysis stems from the recently observed unusual sensitivity of the IP and DIP model to the design of the computational mesh (see Section 2.2). Such oversensitivity seems never to have been observed before (see e.g. Schubert and Sanders, 2012 for a successful field scale application of the IP model using different mesh resolutions and different porosity parametrization methods). It is not observed with the SP and MP models (Guinot, 2012; Guinot and Soares-Frazão, 2006; Soares-Frazao et al., 2008), that use identical storage and connectivity porosities. This leads to wonder whether the oversensitivity of the IP/DIP model to grid design arises from the dual definition (domain- and boundary-based) of porosity or from specific features of the mesh design.

This paper is organised as follows. In Section 2, the oversensitivity of the IP/DIP model to mesh design is illustrated by a simple computational example. Such oversensitivity is explained by a two-dimensional consistency analysis. In Section 3, a two-dimensional characteristic analysis is carried out for the IP and DIP models. It is illustrated with numerical examples in Section 4. Section 5 provides guidelines for the design of IP/DIP meshes and conclusions.

## 2. Consistency analysis of the IP/DIP models

### 2.1. Overview of the models

The governing equations for the Integral Porosity (IP) (Sanders et al., 2008) and Dual Integral Porosity (DIP) (Guinot et al., 2017) models are obtained by applying mass and momentum balances to a control volume $\Omega$ with boundary $\Gamma$

$$\partial_t \int_\Omega \phi_\Omega h \mathrm{d}\Omega + \int_\Gamma \phi_\Gamma \mathbf{q}_\Gamma . \mathbf{n}\, \mathrm{d}\Gamma = 0 \tag{1a}$$

$$\partial_t \int_\Omega \phi_\Omega \mathbf{q} \mathrm{d}\Omega + \int_\Gamma \phi_\Gamma \left[ (\mathbf{q}_\Gamma . \mathbf{n}) \mathbf{q}_\Gamma + \frac{g}{2} h_\Gamma^2 \mathbf{n} \right] \mathrm{d}\Gamma = \int_\Omega \mathbf{s}_\Omega \mathrm{d}\Omega + \int_\Omega \mathbf{s}_\Gamma \mathrm{d}\Gamma \tag{1b}$$

where $g$ is the gravitational acceleration, $h$ and $h_\Gamma$ are respectively the water depth over $\Omega$ and $\Gamma$, $\mathbf{n}$ is the outwards normal unit vector to the boundary, $\mathbf{q}$ and $\mathbf{q}_\Gamma$ are the unit discharge vectors over $\Omega$ and $\Gamma$, $\phi_\Omega$ and $\phi_\Gamma$ are respectively the storage and connectivity porosity, $\mathbf{s}_\Omega$ is the momentum source term arising from the bottom slope and friction onto the bottom, $\mathbf{s}_\Gamma$ is the momentum source term arising from energy dissipation due to building drag and the reaction to the pressure force exerted by the building walls onto the water. The detailed expression for $\mathbf{s}_\Omega$ and $\mathbf{s}_\Gamma$ can be found in Sanders et al. (2008). It is not important at this stage because the present study focuses on the wave propagation properties of the model, in situations where the source terms are zero. In what follows, the following assumptions are thus retained: horizontal, frictionless bottom and negligible building drag forces.

In the IP model (Sanders et al., 2008), the following closure is assumed between the domain and boundary variables:

$$h_\Gamma = h, \quad \mathbf{q}_\Gamma = \mathbf{q} \tag{2}$$

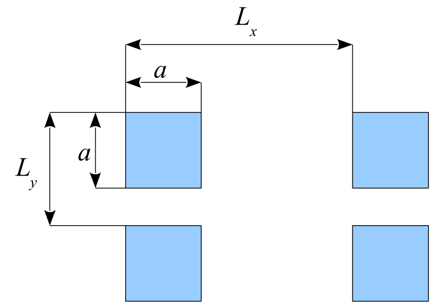


**Fig. 1.** Periodic, idealized urban layout. Definition sketch. Only one period is shown in each direction of space.

The closure introduced in the DIP model is shown in Guinot et al. (2017) to provide a better upscaling of the shallow water equations:

$$h_\Gamma = h, \quad \mathbf{q}_\Gamma = \frac{\phi_\Omega}{\phi_\Gamma} \mathbf{q} \tag{3}$$

This closure model is shown in Guinot et al. (2017) to have a strong influence on the wave propagation properties of the solutions. However, the analysis in Guinot et al. (2017) is restricted to one-dimensional flow configurations.

### 2.2. Oversensitivity to grid design: a simple example

Consider an idealized urban layout made of square house blocks of identical size, regularly spaced along the $x-$ and $y-$directions (Fig. 1). Let $a$, $L_x$ and $L_y$ be respectively the block width and the $x-$ and $y-$spatial periods of the urban layout. Using the IP and DIP models require that a storage and connectivity porosity be defined for this layout. The storage porosity is defined as the fraction of space available to water storage, that is $\phi_\Omega = 1 - \frac{a^2}{L_x L_y}$.

According to Sanders et al. (2008), the definition of the connectivity porosity is not unique and depends on the meshing strategy used to solve the IP equations numerically. Fig. 2 shows three possible mesh designs. In the first (Fig. 2a), rectangular cells are defined from the centroids of the building blocks. The connectivity porosity is $\phi_1 = 1 - \frac{a}{L_y}$ along the vertical edges and $\phi_2 = 1 - \frac{a}{L_x}$ along the horizontal edges. In the second mesh design (Fig. 2b), the computational cells are parallelograms with corners located at the centroids of the blocks. The connectivity porosity is $\phi_1$ along the vertical edges and $\phi_2$ along the diagonal edges (assuming $L_x > L_y$). The third mesh design (Fig. 2c) is the union of the previous two, which results in right-angled triangular cells whose corners are again the centroids of the house blocks. In this design, the connectivity porosity is $\phi_1$ along the vertical edge and $\phi_2$ along the horizontal edge. Along the hypotenuse, it is $\phi_2$ if $L_x > L_y$ and $\phi_1$ otherwise.

Mesh designs 1 to 3 are used to simulate the propagation of a wave into a semi-infinite building layout using the IP model. The initial and boundary conditions are illustrated in Fig. 3. The bottom is flat, motion is assumed frictionless. The water is initially at rest, at a depth $h = h_0$ and a zero velocity at all points. The boundary condition is a zero mass flux across the Western boundary, except over a region of length $L$, where the constant depth $h = h_1 \neq h_0$ is prescribed from $t = 0$ onwards. A wave is generated and propagates into the domain. The semi-infinite domain is simulated by generating a large mesh and stopping the simulation before the wave reaches the mesh boundaries. The governing equations are solved using a finite volume procedure detailed in Sanders et al. (2008). The fluxes are computed using a modified HLLC Riemann solver (Guinot et al., 2017).

The parameters of the test case are given in Table 1.