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Multiscale mimetic method for two-phase flow in fractured media using embedded discrete fracture model

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ABSTRACT

A multiscale mimetic method is developed for the simulation of multiphase flow in fractured porous media in the context of an embedded discrete fracture model (EDFM). The EDFM constructs independent grids for matrix and fracture system. Therefore, it is an efficient and practical flow model as it avoids the complicated unstructured grid subdivision and computing process. In order to extend the EDFM to field-scale applications, we integrate EDFM into a multiscale mimetic method. In this work, we use the multiscale basis functions to capture the detailed interactions between the fractures and the background. The multiscale basis functions are calculated numerically by solving EDFM on the local fine-grid with mimetic finite difference (MFD) method. The MFD method is conservative and robust, which makes it possible to deal with highly complex grid systems. Through combination of multiscale mimetic method and EDFM, this formulation can generate accurate velocity field and pressure field on the fine-scale grid more efficiently than the traditional methods. Numerical results are presented for verification of this multiscale mimetic approach for embedded discrete fracture media, and demonstrate its computational efficiency. The results show that this method is an accurate and efficient method for flow simulation in real-field fractured heterogeneous reservoirs.

1. Introduction

Lots of engineering problems, such as reservoir development, groundwater transport, geothermal exploitation, largely depend on the information provided by simulation of flow through porous media. Apart from their intrinsic heterogeneous properties, these geological formations commonly contain complex fractures, with multiple scales and different conductivity properties. Given their essential influence on flow patterns, the fractures should be represented accurately.

Among the methods targeted at describing fractures explicitly, single-porosity model (Ghorayeb and Firoozabadi, 2000) regards fractures as a narrow high-permeability region. Fully discrete single-porosity models may contain up to 100 million grid cells. Hence, the simulation of such models is deemed intractable even with the advent of supercomputers. Discrete fracture model (DFM) (Huang et al., 2011a, 2011b; Karimi-Fard and Firoozabadi, 2001; Hauge and Aarnes, 2009; Hoteit and Firoozabadi, 2008) is based on conforming unstructured grid generation techniques which treat fractures as inner boundaries of the matrix cells. However, this technique would cause difficulties in grid generation and computing progress. Especially when the distances between fractures are small, the grid generation often has a poor quality which will lead to miscalculation. As an alternative, the embedded discrete fracture model (EDFM) (Lee et al., 2001; Li and Lee, 2008; Yan et al., 2016) constructs independent grids for matrix and fracture system. That is, the discrete fracture network is embedded into the matrix structured grid system directly. Matrix and fractures are coupled by transfer functions. Therefore, EDFM is appealing for its ability to bypass the challenges related to unstructured grid (Moinfar et al., 2012; Zhou et al., 2014).

Even though EDFM is an efficient flow model due to its ability to avoid the complicated unstructured grid subdivision and computing process, its ability for field-scale simulation is limited by numerical methods used to solve the flow model. Even after homogenization of small-scale fractures, the remaining degrees of freedom still exceed the controllable level of traditional numerical methods. This issue motivated the development of multiscale methods for EDFM.

The multiscale methods discussed herein originate from the seminal paper (Hou and Wu, 1997; Efendiev and Wu, 2002), in which a governing equation with zero right hand side (i.e., homogeneous problem) is solved in each element to construct basis functions. It is similar to the ideas proposed earlier by Babuska et al. (1992) and Babuska and Osborn (1981). Then this

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idea has been expanded by many researchers and spawned a series of relevant methods, including mixed multiscale finite-element (MsMFEM) (Chen and Hou, 2003; Aarnes, 2006, 2005), Generalize multiscale finite element method (GMsFEM) (Efendiev et al., 2013, 2015), multiscale finite-volume (MsFV) method (Jenny et al., 2003, 2004; Hajibeygi and Jenny, 2009). All these methods construct basis functions by solving local flow problems to incorporate fine-scale effects into coarse-scale flow equations. Early versions of multiscale methods are proposed as a robust alternative to upscaling methods. Shortly thereafter, these methods have been expanded to model multiphase flow in porous media (Juanes, 2005; Krogstad et al., 2009; Efendiev et al., 2006).

During the past few years, multiscale methods have been extended to simulate fluid flow in fractured media. They were initially proposed to model fractured porous media which treat fractures as narrow high-permeability regions (Natvig et al., 2011; Gulbransen et al., 2009). Later, instead of regarding fractures as volumetric element, discrete fracture model was integrated into multiscale finite element method (Zhang et al., 2016) and GMsFEM (Akkutlu et al., 2015). Recently, a combination of iterative multiscale finite volume method and hierarchical fracture model was presented (Hajibeygi et al., 2011; Matei et al., 2016). These multiscale methods have achieved satisfying results and made great contributions to the exploration of multiscale methods toward fractured porous media.

In this work, we develop a multiscale mimetic method in the context of an embedded discrete fracture model (EDFM). The final matrix formulation of the multiscale mimetic method is similar to the multiscale mixed finite method. However, the discretization of the multiscale mimetic method is similar to finite volume method. Mimetic finite difference (MFD) method builds computational formulation on separate gridcells by introducing pressures located at the middle of the cell faces. This makes MFD could deal with any complex grid.

To summarize, different from existing multiscale formulations, by combing multiscale mimetic method and EDFM, this method could avoid the complicated unstructured grid generation and computing process. Moreover, as discussed in many works (Skaflestad and Krogstad, 2008; Aarnes et al., 2008, 2014), multiscale mimetic method can deal with highly complex grid systems because of its applicability for the complex unstructured grid. Besides, the excellent local conservation property of MFD method makes multiscale mimetic method produce conservative velocity fields which are necessary for flow simulation.

This paper proceeds as follows: we start by introducing the embedded discrete fracture model and its discretization. Next, we describe the multiscale mimetic method briefly and then extend it to deal with the EDFM system. In the numerical experiment section, several 2D and 3D numerical test cases are presented to validate the correctness and effectiveness of the multiscale method. Finally the concluding remarks were given in the final section.

2. Embedded discrete fracture modelling

2.1. Mathematical model

The flow system for incompressible and isothermal two-phase flow without considering the influence of gravity in fractured media is governed by pressure and saturation equations. The pressure equations are written as

Matrix system:

$$-\nabla\cdot(\boldsymbol{K}_{\mathrm{m}}\lambda_{\mathrm{m}}\cdot\nabla p_{\mathrm{m}}) = q_m + \frac{q_{\mathrm{mf}}}{V_{\mathrm{m}}}\delta_{\mathrm{mf}} \tag{1}$$

Fracture system:

$$-\nabla\cdot(\boldsymbol{K}_{\mathrm{f}}\lambda_{\mathrm{f}}\cdot\nabla p_{\mathrm{f}}) = q_{\mathrm{f}} - \frac{q_{\mathrm{mf}}}{V_{\mathrm{f}}} - \frac{q_{\mathrm{ff}}}{V_{\mathrm{f}}}\delta_{\mathrm{ff}} \tag{2}$$

Here, $\mathbf{K}_i(i=m, f)$ stands for the permeability tensor; $p_{i,}$ V_{i} represent pressure, volume of grid cells, respectively; $q_i = q_{\mathrm{in}} + q_{\mathrm{iw}}$ is sink/source term (the subscript w denotes the wetting phase and n denotes the non-wetting phase); $\lambda_i = \lambda_{\mathrm{in}} + \lambda_{\mathrm{iw}}$ is the total mobility where $\lambda_{i\alpha} = k_{ri\alpha}/\mu_\alpha$ is the mobility of phase α, which depends on relative permeability$k_{ri\alpha}$ and viscosity μ_α; q_{mf} is transfer flow between matrix and fracture; q_{ff} is transfer flow between intersecting fractures; $\delta_{\mathrm{mf}}=1$ if matrix cell contains fracture cells, else $\delta_{\mathrm{mf}}=0$; $\delta_{\mathrm{ff}}=1$ if fracture cell intersects with another fracture cells, else $\delta_{\mathrm{ff}}=0$.

The pressure in matrix is assumed to be continuous, therefore, the crossflow term can be written in this form

$$q_{\mathrm{mf}} = -T_{\mathrm{mf}}(p_m - p_{\mathrm{f}}) \tag{3}$$

Here, $T_{\mathrm{mf}} = k_{\mathrm{mf}}\lambda_{\mathrm{mf}}A_{\mathrm{mf}}/d$, where A_{mf} is interfacial area, d is equivalent distance between matrix cell and fracture cell, λ_{mf} is total mobility decided by upstream weight method, and k_{mf} is harmonically averaged permeability.

The crossflow term q_{ff} of intersected fractures (Fig. 1) is given by (Karimi-Fard et al., 2004)

$$q_{\mathrm{ff}} = T_{\mathrm{ff}}\left(p_{\mathrm{fi}} - p_{\mathrm{fj}}\right) \tag{4}$$

Here, $T_{\mathrm{mf}} = \lambda_{\mathrm{ff}}T_{\mathrm{fi}}T_{\mathrm{fj}}/(T_{\mathrm{fi}} + T_{\mathrm{fj}})$, where $T_{\mathrm{fi}} = k_{\mathrm{fi}}d_{\mathrm{fi}}/\widehat{d_i}$, $T_{\mathrm{fj}} = k_{\mathrm{fj}}d_{\mathrm{fj}}/\widehat{d_j}$; λ_{ff} is total mobility decided by upstream weight method; d_{f} and k_{f} stand for fracture aperture and permeability respectively; $\widehat{d}$ represents the average normal distance between the centre of fracture segments and the intersections of fractures.

The saturation equations are written as:

Matrix system

$$\phi_{\mathrm{m}}\frac{\partial S_{mw}}{\partial t} + \nabla\cdot\boldsymbol{v}_{mw} = q_{mw} + \frac{q_{\mathrm{mf}w}}{V_m}\delta_{mf} \tag{5}$$

$$\boldsymbol{v}_{mw} = f_{mw}[\boldsymbol{v}_m + \boldsymbol{K}_m\lambda_{mn}\cdot\nabla p_{mc}] \tag{6}$$

Fracture system

$$\phi_{\mathrm{f}}\frac{\partial S_{fw}}{\partial t} + \nabla\cdot\boldsymbol{v}_{fw} = q_{fw} - \frac{q_{\mathrm{mf}w}}{V_f}\delta_{mf} - \frac{q_{ffw}}{V_f}\delta_{ff} \tag{7}$$

$$\boldsymbol{v}_{fw} = f_{fw}[\boldsymbol{v}_f + \boldsymbol{K}_f\lambda_{fn}\cdot\nabla p_{fc}] \tag{8}$$

Here, ϕ_i denotes the porosity, $f_{\mathrm{iw}} = \lambda_{\mathrm{iw}}/\lambda_i$ denotes the fractional flow function, and p_{ic} is the capillary pressure.

In this work, considering the computational efficiency, we use IMPES strategy to solve the coupled system. IMPES means the flow equations are solved implicitly to obtain the velocity field. Then the velocity field is employed to solve the saturation equations.

2.2. Discretization

Matrix grid and fracture grid are constructed to solve the flow equations (Fig. 2). The fracture grid is embedded into matrix grid independently. Therefore, this method can alleviate the gridding complexities.

To efficiently handle polyhedral cells, MFD is used as our discretization method. As shown in Fig. 3, A_k is the interface between the matrix cells, $\boldsymbol{n}_k$ is the normal to the interface, $\boldsymbol{x}_{ik}$ is the vector pointing from cell centroid to the interface centroid.

Let $\boldsymbol{v}^f_{\mathrm{m}i} = [v_{\mathrm{m}i1}, v_{\mathrm{m}i2}, \cdots, v_{\mathrm{m}in}]^{\mathrm{T}}$ be the fluxes of interfaces of Ω_i, $p^e_{\mathrm{m}i}$ and $p^f_{\mathrm{m}k}$ is the average pressures located at the gridcell centres and interfaces respectively. These quantities are related through a transmissibility matrix$\boldsymbol{T}_{\mathrm{m}i}$, that is