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Using modal decompositions to explain the sudden expansion of the mixing layer in the wake of a groyne in a shallow flow

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ABSTRACT

The sudden expansion of the mixing layer created in the wake of a single groyne is investigated using Particle Image Velocimetry (PIV). In the region of the sudden expansion a patch of high Reynolds shear stresses are observed. Using low-order representations, created from a Dynamic Mode Decomposition and a search criteria based on a Proper Orthogonal Decomposition, the spatio-temporal mechanism of the sudden expansion is investigated. The present study demonstrates the sudden expansion is created by the periodic merging of eddies. These eddies originate from the upstream separation and the tip of the groyne and merge with recirculating eddies created, downstream of the groyne, at the interface of the mixing layer and the lateral wall.

1. Introduction

In natural flows such as rivers and estuaries, groynes are installed to prevent bank scouring (Przedwojski, 1995; Sukhodolov et al., 2002) or to create and enhance fish habitats (Grift et al., 2003). The majority of these natural flows are bounded flows in a domain for which two dimensions, namely that in the direction of the flow, as well as one traverse dimension, greatly exceed the third dimension, consequently they fulfil definition of a shallow flow (Jirka, 2004).

Dependent upon the magnitude of the transversal velocity gradient, a topographical obstruction of any flow can lead to the formation of a mixing layer. In contrast to in deep flows, the large-scale coherent turbulent structures which populate the far field of a shallow mixing layer, can almost extend the whole depth of the flow. As a consequence these quasi-two-dimensional coherent structures (Q2CS), and their spatio-temporal dynamics, are easily influenced by bed-friction (Chu and Babarutsi, 1988; Nadaoka and Yagi, 1998; Socolofsky and Jirka, 2004; Uijttewaal and Tukker, 1998). From an environmental perspective these Q2CS are of great significance, as their spatio-temporal behaviour governs mass and momentum exchange. Some examples of this can be found in their key role to predict the concentration of pollutants, nutrients and the rates of sediment transport (Boyer et al., 2006; Cheng and Constantinescu, 2014; Rhoads and Sukhodolov, 2004; Sukhodolov and Rhoads, 2001).

Many previous works have investigated the spatio-temporal dynamics of Q2CS created by plane shear instabilities e.g. (Chu and Babarutsi, 1988; Rhoads and Sukhodolov, 2004; Uijttewaal and Tukker, 1998), and have shown that due to the effects of bed-friction the spread/growth rate of a shallow mixing layer is modified. In an experimental study to investigate the effect of topographical forcing on a shallow flow, Talstra et al. (2006) found that unlike in a deep flow, (Armaly et al., 1983), the shallow flow mixing layer bound a second counter rotating recirculation cell. They also found at the downstream edge of the first recirculation cell there was a sudden expansion in the mixing layer. In Talstra (2011) it is suggested that this sudden expansion is caused by the merging of vortex shed from the tip of the obstacle and those associated to the largest downstream gyre. In a more recent study, this sudden expansion was also found to occur when a shallow flow was obstructed by a single lateral groyne, it was also observed that the length of reattachment of the mixing layer with the wall was protracted compared with the a deep case (Safarzadeh and Brevis, 2016; Safarzadeh et al., 2016). A number of time-averaged experimental studies have previously investigated this case, but have neither observed or explained this phenomena (Ahmed et al., 2010; Duan et al., 2009; Francis et al., 1968; Kadota and Suzuki, 2010). The Reynolds Averaged Navier–Stokes (RANS k-ω) simulations of Chrisohoides et al. (2003) observed the dual cell system and found that it was stable but periodically horizontally expand-

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ing and contracting. They further found that eddies from upstream and shed from the tip of the obstacle were engulfed by the second downstream recirculation cell, and as the mixing layer reattached with the lateral wall, vorticity was injected back upstream; however they did not observe a sudden expansion the mixing layer. Safarzadeh and Brevis (2016) recently explained that due to the anisotropy associated to the flow system, RANS simulations, based on isotropic closure models, will not be able to simulate the expansion of the mixing layer or predict the length of reattachment. Other Computational Fluid Dynamics (CFD) studies relating to a single groyne have only focused on the turbulent mechanisms upstream and in the near wake of the groyne e.g. (Garde et al., 1961; Koken, 2011; Koken and Constantinescu, 2008; 2009; Paik and Sotiropoulos, 2005), whilst not directly related to this study it is inferred that these complexities have implications on the dynamics of the mixing layer downstream. From all of these findings it is clear that the dynamics of a shallow mixing layer produced in the downstream wake of a single groyne are complex and governed by a number of factors.

The main goal of this research is to investigate the spatio-temporal mechanisms and Q2CS relating to the sudden expansion of the mixing layer. Whilst, the occurrence of this has been previously observed, the physics leading to it has not. Such an interpretation of the physics is important, as this understanding will help one to hypothesise how different flow and boundary conditions will affect the formation and dynamics of the sudden expansion. This is particularly important from an environmental perspective as the increased moment fluxes relating to this phenomenon can lead to enhanced scouring/mixing processes. To investigate these mechanisms an experimental Particle Image Velocimetry (PIV) study is undertaken. To describe the spatio-temporal mechanism a low-order reconstruction of the flow is made from the temporally orthogonal Dynamic Mode Decomposition (DMD) (Rowley et al., 2009; Schmid, 2010) using on a search criteria based on a spatially orthogonal Proper Orthogonal Decomposition (POD) (Aubry, 1991; Berkooz et al., 1993).

2. Experimental setup

An experimental investigation was undertaken at the Karlsruhe Institute of Technology in a tilting 18 m × 1.82 m shallow water flume, a sketch of which is shown in Fig. 1. The flume's bottom and sidewalls are coated in a black plastic laminate. A single obstacle constructed from aluminium plate measuring 0.25 × 0.05 × 0.05 m was placed perpendicular to and at the wall of flume 12 m from the channel entrance. The flow rate Q was set to 13.5 ls^{-1}, and the flume slope inclined to 0.1 mm/m with a water depth H=0.04 m. The Reynolds number was set at, $\mathrm{Re} = \rho U_0 H/\nu$=29,680, where U_0, ρ and ν are the bulk flow velocity, water density, dynamic viscosity, with a sufficiently low Froude number, $\mathrm{Fr} = U_0/\sqrt{gh} = 0.29$, where g is the acceleration of gravity, to ensure minimal surface disturbances as shown by Uijttewaal (2005). To capture the large-scale turbulent structures, a planar Particle Image Velocimetry (PIV) measurement system was used. The PIV system consisted of an industrial grade camera with a 1200 × 1200 CCD-sensor with 12 bit resolution. The flow was seeded with floating 2.5 mm hexagonal polyester particles using a pneumatic particle dispenser. In a shallow flow floating particles have previously been shown to be effective in capturing the large scale turbulent motions by Weitbrecht et al. (2002). The camera was mounted directly above the water surface at a height of 1.5 m and was set to capture an area of 150 × 45 cm with 5 cm overlapping between upstream and downstream images. Measurements were conducted in eight consecutive planes, one upstream of the obstacle and seven downstream. In each position snapshots were recorded at an acquisition frequency of 37 Hz. The image sequence was analysed using the PIV package for Linux GPIV (van der Graaf, 2010), using multi-pass and image deformation techniques. As discussed by Huang et al. (1997) digital PIV often has an associated error, this error can be associated to the seeding distribution. The work of Higham et al. (2016) also shows outlier vectors can increase this error. In the present study it is estimated these errors are approximately 4%. To reduce the influence of the outlier error the PODDEM algorithm (Higham et al., 2016) was implemented.

3. Proper orthogonal decomposition

POD is a linear statistical method commonly used in fluid mechanics for the extraction and analysis of energy meaningful turbulent structures (Aubry, 1991; Berkooz et al., 1993). POD was independently derived by a number of individuals, and consequently takes a variety of names in different fields such as Karhunen-Loève Decomposition, Singular Value Decomposition (SVD) and Principal Components Analysis (PCA) (Karhunen, 1946; Kosambi, 1943; Loève, 1945; Obukbov, 1954; Pougachev, 1953). A POD extracts the most relevant modes from set of a stochastic signal. In fluid mechanics a POD is typically applied to a set of snapshots, which can be obtained from a computer simulation or experimental data typically with two or three components. The POD is used to determine spatially orthogonal energy relevant structures from statistically steady-state turbulent fields, within a finite time domain, ordering them by their contribution to the total variance of the physical property being analysed, e.g. intensity (Brevis and García-Villalba, 2011). A set of $t = 1, 2, \ldots, T$ temporally ordered snapshots, $\mathbf{V}(x, y; t)$, is considered, each of which is of dimension $X \times Y$. The method requires the construction of an $N \times T$ matrix $\mathbf{W}$ from T columns, $\mathbf{w}(t)$ of length $N = XY$, each one corresponding to a column-vector version of a transformed snapshot $\mathbf{V}(x, y; t)$. A POD can be obtained by :

$$\mathbf{W} \equiv \mathbf{\Phi}\,\mathbf{S}\,\mathbf{C}^{*} \tag{1}$$

where $\mathbf{S}$ is the singular valies matrix of dimension $\Theta \times \Theta$, (Θ are the number of modes of the decomposition, and $(\cdot)^{*}$ represents a conjugate transpose matrix operation). The $\lambda = \mathrm{diag}(\mathbf{S})^2/(N-1)$ is the vector containing the contribution to the total variance of each mode. The elements in λ are ordered in descending rank order, i.e. $(\lambda_1 \geq \lambda_2 \geq \cdots \lambda_\Theta \geq 0)$. In practical terms the matrix $\mathbf{\Phi}$ of dimension $N \times \Theta$ contains the spatial structure of each of the modes and the matrix $\mathbf{C}$ of dimension $\Theta \times \Theta$ contains the coefficients representing the time evolution of the modes. Furthermore, the percentage turbulent kinetic energy contribution, E, of a each POD mode can be obtained via:

$$E(\%) = \frac{\lambda_i}{\sum_{i=1}^{N}} \cdot 100 \tag{2}$$

4. Dynamic mode decomposition

The Dynamic Mode Decomposition (DMD) algorithm was introduced into fluid mechanics by Schmid (2010) & Rowley et al. (2009), based on a Arnoldi Eigenvalue algorithm suggested by Ruhe (1984). Unlike POD, which is based upon a co-variance matrix, using and Arnoldi approximation the DMD fits a high-degree polynomial to a Krylov sequence of snapshots, which are assumed to become linearly dependent after a sufficient number of snapshots (Mezić, 2005; Schmid et al., 2009). As discussed by Schmid (2011) such a representation of a nonlinear process by a linear sample-to-sample map can be closely linked to the concept of a Koopman operator, an analysis tool for dynamical systems. For complex flow systems containing superpositions of turbulent structures and mechanisms, the DMD algorithm can be used to extract spatial modes with single 'pure' frequencies. There are currently