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A terrain-following model of wave boundary layers

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ABSTRACT

Over a variable seabed, conventional boundary layer approximations are rendered to be inadequate because of the large variations in bed elevation in the direction of wave propagation. Applying the method of conformal transformation to map the flow domain with a corrugated boundary onto a uniform strip, we put forward a terrain-following modeling approach for Stokes boundary layer flows, accompanying the recent development of the exact Floquet theory of water waves over a generally periodic seabed. For a non-steep seabed profile, but not necessarily small undulation height compared with the water depth, we solve the vorticity equation to obtain the analytical solutions for the boundary layer velocities, bed shear stress and rate of viscous dissipation, explicitly showing the variations both across the boundary layer and along the bed. For a relatively steep bed profile, a remedy is proposed that allows the velocity profiles to be locally determined across the boundary layer avoiding solving the 2-D differential equation for the vorticity. The modeling methodology is presented here for a constant viscosity, including the case of constant eddy viscosity, but can be extended to the case of variable eddy viscosity to improve turbulence modeling.

1. Introduction

Wave boundary layers at the seabed, also known as the Stokes boundary layers for oscillatory flows, are typically very thin compared with the water depth, ranging from a few millimeters to centimeters depending on the bed and flow conditions (Mei, 1989; Fredsøe and Deigaard, 1992; Nielsen, 1992). The flow structure in this thin layer, however, is of great importance since it determines the bottom shear stress and near-bed turbulent intensity, hence having significant influence on sediment transport, physical, chemical and biological processes near the seabed. The wave boundary layer can also affect the vertical structure of a co-existing current in shallow coastal waters (Fredsøe and Deigaard, 1992). The fluctuating (in space and/or in time) bed stresses and pressure due to wave motion can induce, and interact with, groundwater flows (Mei, 1989; Belibassakis, 2012).

It is generally accepted in the literature that outside the wave boundary layer the potential flow field of wave propagation applies, which by itself is a mathematically challenging problem for generally varying topographies, even assuming the linear dynamics (Rhines and Bretherton, 1973; Athanassoulis and Belibassakis, 1999). Various approximate theories have been developed, manipulating the geometric constraints of seabed profiles, e.g. assuming a gentle slope, small bottom amplitude, or localized abrupt variation such as a step (see Athanassoulis and Belibassakis, 1999, and the references within). When the water wavelength, depth and the scale of bed variations are comparable, numerical or semi-numerical methods are generally sought. The computational costs can be significant even with the restriction to linear waves, depending on the complexity of seabed geometry. This has motivated studies to explore the methods for improvements. For instance, Athanassoulis and Belibassakis (1999) developed the coupled-mode theory to amend the shortcoming in satisfying the bottom boundary conditions in numerical representations, thereby accelerating the numerical convergence. These authors also extended the coupled-mode method to treat oscillatory boundary layer effects over a generally variable bathymetry (Belibassakis and Athanassoulis, 2008). Porter and Porter (2003) formulated a transfer matrix method for wave scattering by a patch of small-amplitude periodic bed, significantly reducing the computation. Most recently, Fokas and Nachbin (2012) proposed a set of non-local equations in the propagation space for weakly nonlinear shallow water waves over a variable bottom using a numerical Schwarz–Christoffel mapping.

For a periodic seabed of arbitrary amplitude and shape, the recent advance in the Floquet theory of linear water waves (Yu and Howard, 2012) has provided a new and general approach that can be regarded as the definitive solution to this type of problems. What distinguishes this theory the most is not that it gives exact solutions but that for any frequency it gives a complete set of linear modes. For 2-D time harmonics motions over a horizontal flat seabed, it is well known that the general mathematical treatment

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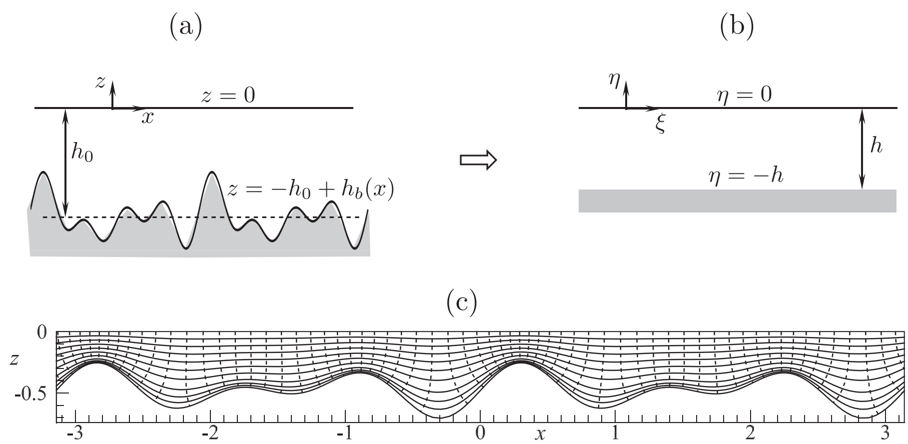

Fig. 1. (a) A 2-D sketch of a homogeneous fluid layer over a periodic seabed, $-\infty < x < \infty$, $-h_0 + h_b(x) \le z \le 0$. (b) The uniform strip of the flow domain upon conformal transformation, $-\infty < \xi < \infty$, $-h \le \eta \le 0$. (c) The orthogonal curvilinear grids in the (x, z) plane that correspond to the Cartesian grids $\xi = constant$ and $\eta = constant$, showing the terrain-following contours near the seabed.

provides a complete basis of solutions: Given a frequency, it consists of two oppositely directed propagating waves and two infinite families of evanescent waves (with rapidly exponentially growing wave amplitudes in opposite directions). This set of flat-bottom linear modes has, for many years, been the only known complete basis for water waves and played a significant role in various engineering and scientific applications. For a periodic seabed, the set of Floquet modes provided in Yu and Howard (2012) is analogous to the above mentioned set of flat-bottom modes and in fact approaches the latter for the same given frequency when the bed undulation height is reduced to zero. Therefore, it becomes the second complete basis for water waves. It has been demonstrated that this set of Floquet modes can be used as a complete basis to construct solutions to boundary-value problems involving a finite extent of periodic bed, similar to the way that the flat-bottom basis is used (Howard and Yu, 2007; Yu and Howard, 2010; Yu and Zheng, 2012; Weidman et al., 2015).

This study is to provide the accompanying boundary layer formulation for linear waves over a periodic bed. Although the nonlinear effects become strong as the wave approaches the shoreline, complete and explicit linear solutions are still very valuable in engineering applications and studies of coastal environments and sustainability. In most practical situations, the wave boundary layer is thin for non-breaking waves and without a resulting rectified current. Even in the case of a turbulent condition, the turbulence is confined to the thin layer just above the bed (Jonsson, 1966; Johns, 1967; Fredsøe and Deigaard, 1992). The flow closely follows the seabed terrain, meaning that the Stokes boundary layer should be measured everywhere in the direction normal to the bed, not in the vertical z direction. However, the conventional boundary layer approximation in the vertical (x, z) plane assumes $\partial/\partial z \gg \partial/\partial x$. This becomes inappropriate when the seabed elevation varies at the scale comparable to the water depth. The conformal transformation developed in Yu and Howard (2012) maps the flow domain over a periodic seabed onto a uniform strip with a flat bottom; see Fig. 1. Since the transformation preserves angles, a vertical distance just above the flat bottom in the mapped plane (ξ, η) corresponds to a normal distance measured from the bed surface in the physical (x, z) plane. Thus, a boundary layer approach formulated in the mapped plane is terrain-following, and the approximations would be made by comparing the derivatives normal and tangent to the bed surface, which respectively measure the variations across the boundary layer and along the bed.

In this study, we assume a constant mean water depth over the general periodic seabed. For many practical applications at scales of engineering interests, this assumption is appropriate since typical beach slopes are gentle (of a few degrees) and the mean water depths do not change significantly. In real coastal regions, a long stretch of periodic (or nearly periodic) seabed is uncommon, but patches of bed undulations of different shapes can occur over a long distance on a shoaling beach. The terrain-following boundary layer approach presented here can be applied locally, considering a finite stretch of periodic seabed. The corresponding potential wave field in such a finite domain can readily be obtained, following the procedure in Yu and Zheng (2012). Here, for the simplicity and clear illustration, we shall only consider a free wave (a Floquet propagating mode) over an indefinitely long periodic bed. For long range propagation of waves, the attenuation of wave energy occurs due to the boundary layer effects. The mathematical tools exist in the literature to deal with long range wave propagation, in particular over a flat or mildly varying seabed (see Mei, 1989 and the references within), and in principle can be adapted to analyze the coupling effects of wave boundary layer and wave amplitude attenuation in the conformally mapped plane. These applications are of scientific and engineering interest, but beyond the scope of the present work and will be explored elsewhere.

Real seabed topographies are in general two-dimensional, but approximately one-dimensional periodic bed features are common (Mei, 1989; Komar, 1998; Pietrzak et al., 1990; Elgar et al., 2003). The theoretical approach presented here can be applied to obtain an accurate first order approximation in those situations. On the other hand, periodic seabeds are often used in laboratory to study wave-topography interactions. The theoretical predictions of the potential wave field and wave boundary layer over a periodic bed are desirable in planning and designing experiments, as well as analyzing data.

Whereas the fully nonlinear and terrain-following wave boundary layer dynamics is described by the vorticity equation in the mapped plane, cf. (3.10), the linearized equation for the Stokes boundary layer (3.13) is based on the assumption of small amplitude waves of the outer potential flows. Neither of these equations makes assumption on the steepness of seabed undulations, nor on the relative importance of the normal and tangential derivatives. They can be solved numerically if needed, matching with the outer potential wave field that can be provided by the Floquet theory (Yu and Howard, 2012) if linear waves are considered or by other wave theories deemed to be appropriate. However, for non-steep topographies (but not necessarily small undulation height compared with the water depth), we can further simply (3.13), obtaining explicit analytical solutions of the boundary layer velocity distribution and bed shear stress. This is the main focus of this study.