Quantifying spatial distribution of spurious mixing in ocean models

Mehmet Ilıcak

Uni Research Climate, Bjerknes Centre for Climate Research, Bergen, Norway

ARTICLE INFO

ABSTRACT

Numerical mixing is inevitable for ocean models due to tracer advection schemes. Until now, there is no robust way to identify the regions of spurious mixing in ocean models. We propose a new method to compute the spatial distribution of the spurious diapycnic mixing in an ocean model. This new method is an extension of available potential energy density method proposed by Winters and Barkan (2013). We test the new method in lock-exchange and baroclinic eddies test cases. We can quantify the amount and the location of numerical mixing. We find high-shear areas are the main regions which are susceptible to numerical truncation errors. We also test the new method to quantify the numerical mixing in different horizontal momentum closures. We conclude that Smagorinsky viscosity has less numerical mixing than the Leith viscosity using the same non-dimensional constant.

1. Introduction

This paper provides a novel method that quantifies the spatial distribution of spurious diapycnic mixing in numerical models of ocean circulation. We update the study of Ilıcak et al. (2012) that employed the reference potential energy methods to measure spurious mixing in ocean general circulation models.

State-of-the-art ocean models have spurious diapycnic (cross-density) mixing due to truncation errors in the discrete tracer advection schemes (Griffies et al., 2000; Ilıcak et al., 2012) and/or cabbeling which is arising from the nonlinearity of the equation of state for seawater (McDougall, 1987; IOC et al., 2010). Ilıcak et al. (2012) showed that all type vertical coordinate ocean models (geopotential, terrain following and isopycnic coordinate) suffer from spurious diapycnic mixing due to cabbeling, while geopotential and terrain following coordinate models suffer from numerical mixing due to advection schemes.

There are various methods that have been used to diagnose spurious mixing in ocean models. Griffies et al. (2000) quantified spurious mixing in idealized basin scale simulations and suggested that it may be significant for large scale ocean climate. Ilıcak et al. (2012) examined four idealized test cases and measured the spurious diapycnic mixing using the evolution of reference potential energy (RPE) proposed by Winters et al. (1995). The idealized test cases are; i) lock exchange, ii) overflow, iii) internal wave, and iv) baroclinic eddies. These cases progress from simple domains and initial conditions to more complex dynamics and topography, so

that spurious mixing may be measured under a variety of conditions. Ilıcak et al. (2012) found that grid Reynolds number and momentum advection scheme are as important as the choice of tracer advection scheme. They concluded that different momentum closures might lead to different amounts of spurious mixing. Recently, Petersen et al. (2015) performed the same test cases with the Model for Prediction Across Scales-Ocean (MPAS-Ocean) using the arbitrary Lagrangian-Eulerian method in the vertical. An alternative approach was suggested by Burchard and Rennau (2008), based on the variance decay induced by the tracer advection scheme to quantify local numerical mixing. However, their method does not distinguish between isopycnic (along-density) and diapycnic mixing. Later, Urakawa and Hasumi (2014) quantified numerical mixing in terms of spurious water mass transformation rates. Despite the progress in developing the diagnostic methods of numerical mixing and dissipation, all the studies reviewed so far, however, could not answer the most important question; *where does spurious mixing occur?*

In this study, we propose a new method that can quantify the spatial distribution of the spurious mixing in an ocean model. We test the new method using the lock-exchange and baroclinic eddies test cases described by Ilıcak et al. (2012) with the MIT general circulation model (MITgcm, Marshall et al., 1997). To our knowledge, this is the first time that a method that computes the location of spurious diapycnic mixing is provided. The aim of this paper is to address the following questions;

1. can the proposed method compute the spatial distribution of spurious mixing? in which regions can spurious diapycnic mixing be high?

E-mail address: mehmet.ilicak@uni.no

2. will different momentum closures make a difference in spurious mixing?

The new method is an extension of the available potential energy density method described by Winters and Barkan (2013). We refer to the new method as the reference potential energy density (RPED) method. We find out that RPED diagnoses the spurious mixing regions to coincide with high-shear regions. Section 2 describes this new method used to diagnose the regions of spurious diapycnic mixing. Sections 3.1 and 3.2 present results and analysis for the idealized test cases consisting of a lock-exchange, and baroclinic eddies in a channel, respectively. We present our conclusions in Section 4.

2. Resting potential energy density method

In this section, we describe the new method. First we describe basic energy terms of a system such as potential energy, available potential energy and reference potential energy. Then we describe available potential energy density method proposed by Winters and Barkan (2013). Finally, we extend their method to compute spurious diapycnic mixing.

The total potential energy (PE) of a fluid is calculated as the volume integral of the density-weighted geopotential:

$$PE(t) = g \iiint \rho z \, dV. \tag{1}$$

Following Winters and Barkan (2013), the available potential energy (APE) is defined in terms of the reference profile $\rho(z^*)$ where z^* is the equilibrium height of a fluid parcel with potential density ρ. This leads to

$$APE(t) = g \iiint \rho (z - z^*) \, dV. \tag{2}$$

The definition of reference potential energy is therefore simply

$$RPE(t) = PE - APE \tag{3}$$

$$RPE(t) = g \iiint \rho z^* \, dV. \tag{4}$$

Winters and Barkan (2013) defined a new term, namely available potential energy density $\mathcal{E}_{APE}$

$$\mathcal{E}_{APE}(\mathbf{x}, t) = (z - z^*)(\rho(\mathbf{x}, t) - \bar{\rho}(z, z^*)), \tag{5}$$

where

$$\bar{\rho}(z, z^*) = \frac{1}{(z - z^*)} \int_{z^*}^{z} \rho(z^{*\prime}) dz^{*\prime}. \tag{6}$$

They showed that the available potential energy density has two properties:

$$g \iiint \mathcal{E}_{APE}(\mathbf{x}, t) dV = APE(t) \tag{7}$$

$$\mathcal{E}_{APE}(\mathbf{x}, t) \geq 0 \qquad \forall \mathbf{x}, t. \tag{8}$$

We can re-write the definition of APE using Eqs. (5) and (7) as

$$APE(t) = g \iiint (z - z^*)\rho(\mathbf{x}, t) dV - g \iiint \left(\int_{z^*}^{z} \rho(z^{*\prime}) dz^{*\prime} \right) dV. \tag{9}$$

Denoting the terms on the right hand side (RHS) in Eq. (9) as Γ_1 (the first term on the RHS) and Γ_2 (the second term on the RHS) respectively, Winters and Barkan (2013) proved that global sum of Γ_2 is equal to zero in the domain. Thus, Eq. (9) recovers the original equation described above (Eq. (2)). Winters and Barkan (2013) also defined Γ terms for each ith parcel with volume element dV_i located at height z_i with density ρ_i as

$$\Gamma_1^i = g dV_i \rho_i (z_i - z_i^*) \tag{10}$$

$$\Gamma_2^i = -g\Delta z^* \sum_{j=1}^{n} \rho_j dV_i, \tag{11}$$

where n is the number of dV parcels with height in the reference density profile between z_i^* and z_i. The z_i^* term is the equilibrium position of the parcel ith (i.e. $z_i^* = z^*(\rho_i)$), and the increment Δz^* is a signed quantity with a magnitude $dz^* = dV/A$. Winters and Barkan (2013) employed $\mathcal{E}_{APE}$ to compute a three dimensional structure of available potential energy in the system (see their Fig. 5).

Here we extend their work by defining a new quantity called reference potential energy density, $\mathcal{E}_{RPE}$, as

$$\mathcal{E}_{RPE}(\mathbf{x}, t) = z^* \rho(\mathbf{x}, t) + \int_{z^*}^{z} \rho(z^{*\prime}) dz^{*\prime}. \tag{12}$$

The second term in Eq. (12) is the opposite sign of second term in Eqs. (5) and (9). This new quantity has the property of $g \int \mathcal{E}_{RPE}(\mathbf{x}, t) dV = RPE(t)$ which leads to

$$RPE(t) = g \iiint z^* \rho(\mathbf{x}, t) dV + g \iiint \left(\int_{z^*}^{z} \rho(z^{*\prime}) dz^{*\prime} \right) dV, \tag{13}$$

$$RPE(t) = \Gamma_3 + \Gamma_2. \tag{14}$$

It can be easily shown that sum of Eqs. (9) and (13) is equal to the total potential energy, PE (Eq. (1)).

The reference potential energy density is a 4D field (x,y,z,t) and the change of $\mathcal{E}_{RPE}$ over a time increment $(t, t + \Delta t)$ shows regions where absolute reference potential energy increased in the system due to diapycnic mixing. Since the diapycnic mixing is irreversible, absolute reference potential energy change has to be positive definite. The only caveat is that we also need to take into account advection of $\mathcal{E}_{RPE}$ in time;

$$\frac{d\mathcal{E}_{RPE}}{dt} = \frac{\partial \mathcal{E}_{RPE}}{\partial t} + \mathbf{u} \cdot \nabla_H \mathcal{E}_{RPE} + w \frac{\partial \mathcal{E}_{RPE}}{\partial z}, \tag{15}$$

where d/dt is the material derivative, $\mathbf{u}$ is the two dimensional horizontal velocity, ∇_H is the horizontal gradient operator and w is the vertical velocity (see Appendix).

We present results from this new method in the next section. The advantages of this new method are i) it works on energetics rather than "effective" diffusivity, ii) it can be implemented both on line and off line which saves computational time, iii) it shows the regions of spurious and explicit mixing.

3. Results

We show results from two different model experiments; i) 2D lock-exchange ii) 3D baroclinic eddies in a periodic channel. We test the new $\mathcal{E}_{RPE}$ method using the MIT general circulation model (MITgcm). The MITgcm is a three dimensional C-grid ocean code using fully incompressible (the Boussinesq approximation) Navier Stokes equations (Marshall et al., 1997). Here we employ the MITgcm with the z−level coordinate formulation, a linear equation of state and the hydrostatic approximation. In addition, convective adjustment and explicit diffusion parameterizations are turned-off, if it is not mentioned otherwise.

3.1. Lock-exchange

The model configuration is similar to the non-rotating gravitational adjustment problem described in Haidvogel and Beckmann (1999) and Ilıcak et al. (2012). The computational domain is two-dimensional, with horizontal dimension $0 \leq x \leq L$ and vertical dimension $-H \leq z \leq 0$, where $L = 64$ km and $H = 20$ m. Insulated boundary conditions are used for temperature and salinity on all