Ocean Modelling 117 (2017) 1–11

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Topographic and frictional controls on tides in the Sea of Okhotsk

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ARTICLE INFO

Article history:
Received 23 March 2017
Revised 30 May 2017
Accepted 25 June 2017
Available online 27 June 2017

Keywords:
Tide
Adjoint sensitivity
Bottom topography

ABSTRACT

The sensitivity of barotropic tides to bottom topography and frictional parameters has been studied in a model for the Sea of Okhotsk. This region was chosen because of the paucity of bathymetry data and the possibility of using satellite altimeter data to better identify the bottom topography using variational inverse methods. The sensitivity was studied using both the direct and adjoint sensitivity. In the former approach, perturbations to the nominal model were applied to examine their impact; in the latter approach, the sensitivities were computed using the adjoint of the tangent linearization of the dynamical model. It is found that small-scale coastal near-resonant amplification controls the tidal dynamics, and the sensitivity of the solutions is dominated by topography in these regions, far exceeding the influence of other factors. Consequently, the tidal dynamics and resonant amplification creates a non-local relationship between water level and bottom topography and leads to a linear dependence of measurements upon a very few degrees of freedom. The results indicate severe limitations on inverse approaches for identification of topography, and add to the rationale for the collection and sharing of high quality bathymetry data to enable improved ocean modeling.

1. Introduction

Numerical modelers of the coastal ocean have long recognized the importance of bottom topography for making accurate models of tides and water levels (Lefevre et al., 2000). The bottom topography is here defined as the two-dimensional field of water depth, i.e., the vertical distance between the mean water surface and the material bottom. The term *bathymetry* is sometimes used to refer to *bottom topography*; however, here the usage of *bathymetry* shall be restricted to refer to measurements of water depth via hydrographic surveys (Organization, 1998), rather than referring to the two-dimensional depth field.

Bottom topography influences the flow fields through several distinct mechanisms. First, bottom topography defines the lower material boundary of the ocean, which provides kinematic constraints on the flow. Second, at a given location, the water depth is related to the mass of the water column and determines the acceleration resulting from a horizontal force; the speed of wave propagation is thus set by water depth. And, finally, the ocean bottom is obviously the locus of boundary layer processes, and these support tangential stresses that may influence entire the water column. Although the detailed representation of these physics differs depending on the setting, their manifestations on wave processes, material transport, vorticity dynamics, and dissipative processes are frequently significant.

It can be a challenge for ocean modelers to obtain the topography needed to create credible ocean simulations. The availability of bathymetry obtained via ordinary or multibeam sonar varies greatly as a function of geographic location (Wessel and Chandler, 2011), and the accuracy of older measurements is not well characterized (Jakobsson et al., 2002; Marks and Smith, 2008). In data from the pre-satellite era, the navigational or geolocation error (20 km being a typical magnitude, Smith 1993) contributes the most to uncertainty in depth, in direct proportion to the slope of the bottom (Jakobsson et al., 2005). At the interface between the continental shelf and slope, the error in depth can be 100's of meters (Marks and Smith, 2005; Inazu et al., 2009). Other sources of error include the uncertain vertical datum, when the measured depth is significantly altered by tides or other time-variable water depth which is not corrected, error due to the non-vertical orientation of the sonar beam, and error in the speed of sound used to convert travel time to distance (Marks and Smith, 2008). Smith and Sandwell (1994) pioneered the use of the marine gravity field for inferring bottom topography in the deep ocean, where sediments are uniform and the relationship between gravity and topography can be inferred. This approach is less accurate over the continental shelves and regions of complex geology (Marks and Smith, 2012), where dense in situ data are necessary to control the gravimetry-based estimates (Marks et al., 2010; Marks and Smith, 2012).

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http://dx.doi.org/10.1016/j.ocemod.2017.06.011

In order to deal with these uncertainties, it is typical in the course of model development to examine the sensitivity of the solution to changes in the source of the topography and the degree of smoothing, as well as factors such as grid resolution (Hirose et al., 2001). And, sometimes, the topography is calibrated or adjusted to improve the model performance (Ten-Brummelhuis et al., 1993; Lyard and Genco, 1994; Hirose, 2005; Cea and French, 2012).

The present study was motivated by the hypothesis that an inverse modeling approach could usefully improve the accuracy of bottom topography over continental shelves at locations where bathymetry is sparse and gravimetrically-derived topography is inaccurate. The barotropic tides are probably the most-precisely measured and predictable phenomena involving the oceans at continental-shelf scale (Stammer et al., 2014), so the inverse approach was developed by assimilating altimeter-derived water elevation measurements into a dynamical tide model. The technical approach involved re-working the implementation of Zaron et al. (2011) to the frequency domain by augmenting the approach of Egbert et al. (1994) with topography as a distributed control parameter. Poor results with the inverse approach and unexpected sensitivity to the spatial correlation model for the topographic error led to the more fundamental investigation of model sensitivity described in the present article.

This paper is organized as follows. In Section 2, the concepts of the forward and adjoint sensitivity are defined and their application to a barotropic shallow water model of tidal dynamics is reviewed. In Section 3, an overview of the factors influencing the accuracy of tide models in the Sea of Okhotsk is conducted in order to define the relative importance of bottom topography, grid resolution, frictional parameterization, and non-linear tidal interactions. In Section 4, the adjoint sensitivity is used to quantify the sensitivity of tidal elevation to topography and friction, and the linear dependence of the adjoint sensitivity functions is examined. In Section 5, the findings of Section 4 are related to a previous idealized study that emphasized the significance of tidal resonances near the coastline, and other implications of the results are discussed. Section 6 summarizes the results.

2. Methodology

2.1. Tide model and dynamics

The tidal dynamics are expressed in terms of the horizontal volume transport vector, **U**, comprised of zonal and meridional components (*U*, *V*), and the water surface elevation, η, which are functions of latitude, θ, longitude, ϕ, and time, *t*. The time-varying fields (**U**, η) are expanded as the sum of contributions from $k = 1, \ldots, N_c$ astronomical tidal constituents in the present case,

$$\mathbf{U}(\phi,\theta,t) = \sum_{k=1}^{N_c} Re[\mathbf{U}^{(k)}(\phi,\theta)\exp(-i\omega_k t)] \tag{1}$$

$$\eta(\phi,\theta,t) = \sum_{k=1}^{N_c} Re[\eta^{(k)}(\phi,\theta)\exp(-i\omega_k t)], \tag{2}$$

where ($\mathbf{U}^{(k)}$, $\eta^{(k)}$) are complex-valued harmonic constants for the transport and surface elevation fields, and ω_k are the frequencies of tides. In the present case, no more than $N_c = 8$ constituents shall be considered, corresponding to the M_2, K_1, S_2, O_1, N_2, P_1, K_2, and Q_1 tides, for $k = 1, \ldots, 8$, repectively. Henceforth the notation indicating a particular frequency (*k*) shall be omitted except when necessary in expressions involving more than a single frequency.

Tidal dynamics are governed approximately by the Laplace Tidal Equations (LTE) augmented with a linear approximation of the quadratic bottom drag law (Snyder et al., 1979) and a modified astronomical tide generating force (ATGF) which accounts for solid Earth loading and ocean self-attraction (Egbert et al., 1994). The LTE are given by,

$$-i\omega\mathbf{U} + f\times\mathbf{U} + gH\nabla(\eta-\Phi) + C_d u_f\frac{\mathbf{U}}{H} = 0 \tag{3}$$

$$-i\omega\eta + \nabla\cdot\mathbf{U} = 0 \tag{4}$$

where $f = 2\Omega\sin\theta\hat{k}$ is the local vertical component of the Coriolis parameter, $g = 9.81\mathrm{m/s}^2$ is gravitational acceleration, *H* is the bottom topography, $C_d = 2.5\times10^{-3}$ is the bottom drag coefficient, and u_f is an estimate for the time-average near-bottom current speed. The domain, denoted $\mathcal{D}$, has boundary, $\partial\mathcal{D}$, comprising closed (material) and open boundary segments, denoted $\partial\mathcal{D}_1$ and $\partial\mathcal{D}_2$, respectively. Boundary conditions are no-normal flow on $\partial\mathcal{D}_1$,

$$(U,V)\cdot\boldsymbol{n} = 0, \tag{5}$$

and specified surface elevation, η_d, on $\partial\mathcal{D}_2$,

$$\eta = \eta_d. \tag{6}$$

The LTE are thus forced by the modified ATGF, Φ, and by open boundary conditions on tidal elevation η. In spherical polar coordinates the gradient and divergence operators are given by,

$$\nabla\eta = \left(\frac{1}{a\cos\theta}\frac{\partial\eta}{\partial\phi}, \frac{\partial\eta}{a\partial\theta}\right), \tag{7}$$

and

$$\nabla\cdot\mathbf{U} = \frac{1}{a\cos\theta}\frac{\partial U}{\partial\phi} + \frac{1}{a\cos\theta}\frac{\partial(V\cos\theta)}{\partial\theta}, \tag{8}$$

where $a = 6.7308\times10^3$ km is the radius of the Earth.

In order to model the frictional coupling of different frequencies, u_f is computed from the tidal currents, $\mathbf{U}^{(k)}/H$, and non-tidal current, u_0 (Snyder et al., 1979), as

$$u_f = \left(u_0^2 + \gamma_f\frac{1}{2H^2}\sum_k|\mathbf{U}^{(k)}|^2\right)^{1/2}. \tag{9}$$

The optimal value of the coefficient γ_f is related to the frequency content of the tides and may be found by analysis of the quadratic bottom stress (Dronkers, 1964; Snyder et al., 1979; Lavelle and Mofjeld, 1983). Here the values $\gamma_f = 1$ and $u_0 = 0.25$ m/s are used so that u_f is simply the root-mean-square current speed accounting for the resolved tides and a constant non-tidal current. Even with this approximation, the solution of (3)-(6) is iterative, with u_f being evaluated from currents computed at the previous iteration. In practice, 5 iterations are typically used to produce stable estimates of u_f which differ by less than a few percent from previous values.

Note that (**U**, η) depends on tidal frequency (*k*), but *H* and C_d do not depend on (*k*). Thus, in addition to the coupling through u_f, the equations for ($\mathbf{U}^{(k)}$, $\eta^{(k)}$), (3)-(4), are implicitly coupled via *H* and C_d. Additional nonlinearity in the shallow water equations has been omitted from the LTE without detailed justification. Later, in Section 3, some of these nonlinearities shall be examined in more detail by using a specific solution of the LTE to diagnose these terms.

2.2. Forward sensitivity

Suppose the topography and/or drag coefficient are perturbed from background fields, $H = \overline{H} + H'$ and $C_d = \overline{C}_d + C_d'$. Assuming the corresponding background fields, $(\overline{\mathbf{U}}, \overline{\eta}, \overline{H}, \overline{C}_d)$, are solutions of Eqs. (3)-(4), the $(\mathbf{U}', \eta', H', C_d')$ perturbations approximately satisfy the tangent-linearization of the LTE,