



# A comparison of viscous-plastic sea ice solvers with and without replacement pressure



Madlen Kimmritz<sup>a,\*</sup>, Martin Losch<sup>b</sup>, Sergey Danilov<sup>b,c</sup>

<sup>a</sup>Nansen Environmental and Remote Sensing Center and Bjerknes Centre for Climate Research, Bergen, Norway

<sup>b</sup>Alfred Wegener Institute, Bremerhaven, Germany

<sup>c</sup>A. M. Obukhov Institute of Atmospheric Physics RAS, Moscow, Russia

## ARTICLE INFO

### Article history:

Received 6 July 2016

Revised 7 May 2017

Accepted 13 May 2017

Available online 15 May 2017

### MSC:

00-01

99-00

### Keywords:

Sea ice

VP rheology

EVP rheology

MITgcm

Adaptive relaxation parameter

JFNK

Replacement pressure

## ABSTRACT

Recent developments of the explicit elastic-viscous-plastic (EVP) solvers call for a new comparison with implicit solvers for the equations of viscous-plastic sea ice dynamics. In Arctic sea ice simulations, the modified and the adaptive EVP solvers, and the implicit Jacobian-free Newton–Krylov (JFNK) solver are compared against each other. The adaptive EVP method shows convergence rates that are generally similar or even better than those of the modified EVP method, but the convergence of the EVP methods is found to depend dramatically on the use of the replacement pressure (RP). Apparently, using the RP can affect the pseudo-elastic waves in the EVP methods by introducing extra non-physical oscillations so that, in the extreme case, convergence to the VP solution can be lost altogether. The JFNK solver also suffers from higher failure rates with RP implying that with RP the momentum equations are stiffer and more difficult to solve. For practical purposes, both EVP methods can be used efficiently with an unexpectedly low number of sub-cycling steps without compromising the solutions. The differences between the RP solutions and the NoRP solutions (when the RP is not being used) can be reduced with lower thresholds of viscous regularization at the cost of increasing stiffness of the equations, and hence the computational costs of solving them.

© 2017 The Authors. Published by Elsevier Ltd.

This is an open access article under the CC BY-NC-ND license.

(<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

## 1. Introduction

Sea ice covers only approximately 7% of the global ocean, but it is an important contributor to the surface heat budget and hence an important player for the Earth's climate. It undergoes strong annual variations and it is affected by climate change about twice as much as globally averaged quantities (Vancoppenolle, 2008). Thus, for any application in climate sciences, it is important to describe the physics of sea ice accurately. Dynamic and thermodynamic processes determine sea ice evolution. While thermodynamic processes lead to melting and growth of the ice, sea ice dynamics describe the motion and deformation of the sea ice pack under the action of wind forces, ocean currents and internal ice stresses. We focus on the dynamics of sea ice. Most state-of-the-art numerical sea ice model dynamics are based on a quasi-continuum assumption

and treat sea ice as a non-Newtonian fluid with an appropriate formulation of rheology.

The dynamical nature of sea ice is strongly non-linear (Hibler, 1988), mainly due to the strong non-linearity of the internal ice stresses, and encompasses a wide variety of ice types and features. Thus, any realistic rheology for sea ice, that is the relationship between the internal ice stresses and the ice strain rates, leads to a very stiff system of non-linear equations and requires efficient solution methods with good numerical convergence properties.

In spite of recent developments, such as the elastic-plastic-anisotropic (Tsamados et al., 2013) or the elasto-brittle rheology (Girard et al., 2011; Bouillon and Rampal, 2015), the vast majority of sea ice models are based on the viscous-plastic (VP) rheology (Hibler, 1979). To our knowledge, an implicit Jacobian-free Newton–Krylov (JFNK) solver (Lemieux et al., 2010; 2012; Losch et al., 2014) is one of the most efficient way to obtain accurate (machine precision) solutions available today for the highly non-linear VP model, but such a solver is still computationally very expensive. In this manuscript we use converged JFNK solutions as a reference.

\* Corresponding author.

E-mail addresses: [madlen.kimmritz@nersc.no](mailto:madlen.kimmritz@nersc.no), [madlen.kimmritz@awi.de](mailto:madlen.kimmritz@awi.de), [m.kimmritz@yahoo.de](mailto:m.kimmritz@yahoo.de) (M. Kimmritz), [martin.losch@awi.de](mailto:martin.losch@awi.de) (M. Losch), [sergey.danilov@awi.de](mailto:sergey.danilov@awi.de) (S. Danilov).

An alternative is to use fully explicit Elastic-Viscous-Plastic (EVP) schemes in which an elasticity term has been added to the stress equation in order to relax the restrictive time step limitation of VP-models. In this case, sub-cycling within each external time level is applied in order to damp out the artificial elastic waves. The idea (Hunke and Dukowicz, 1997; Hunke, 2001) is now widely used in numerical sea ice modeling. Losch et al. (2010); Losch and Danilov (2012) and Lemieux et al. (2012) showed that the original attempt does not converge to the VP solution, and instead produces different deformation fields, weaker ice and smaller viscosities. To overcome this issue, Lemieux et al. (2012) added an inertial term in the momentum equations. Bouillon et al. (2013) reformulated this modified EVP (mEVP) scheme as a pseudo-time iterative process, which by construction should converge to the VP solution. Kimmritz et al. (2015) formulated a criterion that ensured (linear) convergence of the scheme proposed in Bouillon et al. (2013) in a set of experiments with simple geometry and forcing.

In the mEVP method, two constant sub-cycling parameters  $\alpha$  and  $\beta$  determine the convergence rates of the ice stress and momentum equations to the VP solution in the pseudo-time iteration. They need to be sufficiently large, typically order of several hundreds, to ensure stability of the scheme. Large sub-cycling parameters, however, also mean slower convergence rates and thus likely require a larger number of sub-cycling steps  $N_{\text{EVP}}$  to reach a reasonable degree of convergence. Full convergence (i.e. the residuals of the momentum and stress equations are reduced to machine precision) requires many thousand sub-cycles and has been found to be too expensive to be practical (Kimmritz et al., 2015).

Kimmritz et al. (2016) modified mEVP further and determined the sub-cycling parameters locally according to local stability requirements to ensure sufficient accuracy of the sub-cycling. In this adaptive EVP (aEVP) scheme, the sub-cycling parameters vary in space and time, while the number of sub-cycling steps is kept constant as in the mEVP scheme. The aEVP scheme requires large values for the sub-cycling parameters  $\alpha$  and  $\beta$  only in a few areas where the ice is strong and immobile (Kimmritz et al., 2016). If one accepts poor reduction of residuals in these areas (i.e. low convergence), a smaller overall number of sub-cycling steps can be used without compromising accuracy almost everywhere compared to mEVP.

A practical performance analysis of aEVP and mEVP with realistic ocean geometries and forcing was not a subject of Kimmritz et al. (2016) and is done here. We will show that for both explicit schemes we can reproduce solutions that are nearly indistinguishable (see below) from reference solutions obtained with the converged JFNK solver. Tightly connected to the choice of solution techniques is the practical question of selecting the number of sub-cycling steps  $N_{\text{EVP}}$ . Because running the mEVP and aEVP schemes to full convergence is computationally very expensive, these schemes, in practice, will be run with incomplete convergence. We show that, in order to save computer time,  $N_{\text{EVP}}$  can be reduced well below the value required by formal theoretical consideration with only very limited effect on the obtained solutions.

Another, almost accidental, result emerges that, in contrast to the simple test cases in Kimmritz et al. (2016), the convergence of the mEVP and aEVP schemes to the VP solution and the performance of the JFNK solver in realistic configurations are sensitive to the regularization of the internal ice strength in the viscous regime. Hibler (1979) limited large viscosities for very small strain rates in the internal stress equations by maximal values thereby introducing viscous behavior to the model. Bounding the viscosities from above is almost equivalent to limiting the strain rate parameter  $\Delta$  from below. In some models (including ours), this regularization is implemented by adding a minimum  $\Delta_{\text{min}}$  to  $\Delta$  (see Section 2 for more details) to yield a smooth regularization (Kreyscher et al., 2000). Lemieux et al. (2010) implemented a nar-

rower but still smooth transition from the plastic to the viscous regime by regularizing the viscosities with a hyperbolic tangent (tanh) function. With regularized viscosities, ice strength gradients (i.e., ice thickness and concentration gradients) lead to creep of ice in the absence of forcing. Modifying the compressive strength in analogy to the regularized viscosities removes this spurious effect (Hibler and Ip, 1995). The physical effect of this so-called replacement pressure (RP) on large scale simulations was compared to other rheologies (Geiger et al., 1998), and most, if not all, sea ice models use RP to avoid spurious motion. We re-evaluate the effects of the replacement pressure in the context of numerical convergence of the mEVP and aEVP schemes.

This article is structured as follows. Section 2 describes the sea ice momentum equations followed by a brief introduction of solution methods in Section 3. Section 4 presents the numerical results. A discussion of the results and the conclusions are given in Sections 5 and 6.

## 2. Description of model sea ice dynamics

The dynamics of sea ice is governed by the sea ice momentum balance

$$m(\partial_t + f\mathbf{k}\times)\mathbf{u} = \tau_a + \tau_o - mg\nabla H + \mathbf{F}, \quad (1)$$

where  $m$  is the ice (plus snow) mass per unit area,  $f$  is the Coriolis parameter,  $\mathbf{k}$  the vertical unit vector,  $\mathbf{u}$  the ice velocity,  $\tau_a$  and  $\tau_o$  the wind and ocean stresses,  $g$  the acceleration due to gravity,  $H$  the sea ice surface elevation, and  $\mathbf{F} = \nabla \cdot \sigma$  the divergence of internal stresses in sea ice. In our implementation,  $\tau_a$  is independent of the ice velocities. The ocean stress is prescribed by  $\tau_o = -C_d \rho_o (\mathbf{u} - \mathbf{u}_o) |\mathbf{u} - \mathbf{u}_o|$  with ocean-ice drag  $C_d$ , ocean water density  $\rho_o$  and ocean velocity  $\mathbf{u}_o$ .

The viscous plastic constitutive law is given by

$$\sigma_{ij}(\mathbf{u}) = 2\eta\dot{\epsilon}_{ij} + \left[ (\zeta - \eta)\dot{\epsilon}_{kk} - \frac{P}{2} \right] \delta_{ij} \quad (2)$$

with the strain rates

$$\dot{\epsilon}_{ij} = \frac{1}{2} (\partial_i u_j + \partial_j u_i) \quad (3)$$

where the indices  $i$  and  $j$  denote the  $x$  and  $y$  directions. The ice strength  $P$  is parameterized as  $P = P^* h a e^{-c^*(1-a)}$ , where  $a$  is the ice concentration (or ice compactness) and  $h$  is the mean thickness of the grid cell; the constants  $P^*$  and  $c^*$  are set to  $27500 \text{ Nm}^{-2}$  and  $20$  (Lemieux et al., 2010). The bulk and shear viscosities are given by  $\zeta = P/(2\Delta)$  and  $\eta = \zeta/e^2$ , such that the stress states lie on an elliptic yield curve with the ratio of the semi-major and the semi-minor axis  $e = 2$ . The parameter  $\Delta$  is defined as  $\Delta = (\dot{\epsilon}_d^2 + e^{-2}\dot{\epsilon}_s^2)^{1/2}$  with divergence  $\dot{\epsilon}_d = \dot{\epsilon}_{11} + \dot{\epsilon}_{22}$  and shear  $\dot{\epsilon}_s = ((\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2)^{1/2}$ .

Thus, the ice is presumed to act as a plastic material, unless the shear and the divergence are very small. If the deformation parameter  $\Delta$  is below a given threshold ( $\Delta < \Delta_{\text{min}}$ ), the ice is treated as a linear-viscous fluid. We implement this by replacing  $\Delta$  with  $\Delta_{\text{reg}} = \Delta + \Delta_{\text{min}}$  in the definition of  $\zeta$  and  $\eta$ .

In the case of small strain rates and non-uniform  $P$ , changes in the internal ice stress  $P$  introduce a slow creep towards equilibrium even if no external forces are being imposed. Hibler and Ip (1995) introduced the so called replacement pressure (RP)  $P_r = 2\Delta\zeta = P\Delta/(\Delta + \Delta_{\text{min}})$  to remove this unphysical effect of unforced spontaneous viscous creep. The constitutive law then reads

$$\sigma_{ij}(\mathbf{u}) = 2\eta\dot{\epsilon}_{ij} + \left[ (\zeta - \eta)\dot{\epsilon}_{kk}\delta_{ij} - \frac{P_r}{2} \right] \delta_{ij}. \quad (4)$$

Note, that  $P_r$  is smaller than  $P$  in the viscous regime as the strain rates, and hence  $\Delta$ , tend to zero.

Download English Version:

<https://daneshyari.com/en/article/5766359>

Download Persian Version:

<https://daneshyari.com/article/5766359>

[Daneshyari.com](https://daneshyari.com)