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A deformation-based parametrization of ocean mesoscale eddy reynolds stresses

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A B S T R A C T

Ocean mesoscale eddies strongly affect the strength and variability of large-scale ocean jets such as the Gulf Stream and Kuroshio Extension. Their spatial scales are too small to be fully resolved in many current climate models and hence their effects on the large-scale circulation need to be parametrized. Here we propose a parametrization of mesoscale eddy momentum fluxes based on large-scale flow deformation. The parametrization is argued to be suitable for use in eddy-permitting ocean general circulation models, and is motivated by an analogy between turbulence in Newtonian fluids (such as water) and laminar flow in non-Newtonian fluids. A primitive-equations model in an idealised double-gyre configuration at eddy-resolving horizontal resolution is used to diagnose the relationship between the proposed closure and the eddy fluxes resolved by the model. Favourable correlations suggest the closure could provide an appropriate deterministic parametrization of mesoscale eddies. The relationship between the closure and different representations of the Reynolds stress tensor is also described. The parametrized forcing possesses the key quasi-geostrophic turbulence properties of energy conservation and enstrophy dissipation, and allows for upgradient fluxes leading to the sharpening of vorticity gradients. The implementation of the closure for eddy-permitting ocean models requires only velocity derivatives and a single parameter that scales with model resolution.

1. Introduction

Ocean mesoscale eddies are found throughout the world ocean, and are observed to be especially vigorous in regions of strong western boundary jets such as the Gulf Stream and Kuroshio Extension, as well as throughout the Antarctic circumpolar current. The strength and variability of these ocean jets is strongly enhanced by upgradient momentum fluxes due to mesoscale eddies (Starr, 1968). In order for ocean general circulation models (GCMs) to accurately simulate the mean state and variability of these jets, the effects of mesoscale eddies on the large-scale flow must be represented. Unfortunately, mesoscale eddies have spatial scales in the range 10–100 km, which is too fine to be resolved by the ocean GCMs currently used in coupled climate models and so their effects must be parametrized.

There are broadly two classes of mesoscale eddy parametrization. In GCMs with very coarse horizontal resolution, ≈100 km or coarser, mesoscale eddies are not resolved at all and their effects are parametrized using a scheme that mimics their cumulative effect on the resolved scales of motion, as done by the Gent–McWilliams scheme that represents the flattening of isopycnal surfaces resulting from baroclinic instability (Gent and McWilliams, 1990). In GCMs with slightly finer horizontal resolution (e.g. 50 km), often referred to as eddy-permitting models, mesoscale eddies are partially resolved, but can behave unrealistically because the eddy scale is close to the model grid scale. In this case, the goal of an eddy parametrization is to improve the representation of the eddy variability that is already partially present in the model, and potentially to improve the mean state. Since many of the next generation of coupled climate models will make use of eddy-permitting ocean GCMs, and eddy-resolving GCMs will remain too computationally expensive to be widely used in the near future, it is important to develop mesoscale eddy parametrizations that are suitable for models with eddy-permitting spatial resolutions.

Turbulent mesoscale eddies act to transfer energy from small to large spatial scales as occurs in 2D turbulence, resulting in self-organised upgradient momentum fluxes driving large-scale ocean jets (Charney, 1971). For a numerical model to resolve this be-

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haviour, the important upscale and downscale transfers resulting from the nonlinear (i.e. advective) terms in the equations of motion must be adequately represented. Eddy-permitting models are by definition truncated somewhere in the middle of the range of scales over which such transfers occur, which implies that the effects of parameterized eddy viscosity may be felt strongly within this range because the eddy scales are close to the model gridscale. While it may be necessary for numerical stability to assume that eddies behave diffusively at the smallest resolved scales, eddy viscosity is equivalent to downgradient flux and therefore cannot represent the mean-flow forcing effects of upgradient eddy fluxes.

A more physically consistent parametrization of eddies should account for both upgradient and downgradient momentum fluxes. It should respect the quasi-geostrophic turbulence properties that there are net upscale transfers of energy and net downscale transfers of enstrophy, with eddy-mean interaction leading to the maintenance of large-scale jets. The parametrization should also depend on the variability present in the model, as it is intended for use in eddy-permitting models, and ideally this dependence should have as few as possible adjustable parameters. Our aim here is to design a parametrization that incorporates these properties.

Approaches to eddy parametrization are varied and are often explored in the context of idealised models (e.g. Frederiksen and Davies, 1997; Eden and Greatbatch, 2008; Grooms et al., 2013; Berloff, 2015a; 2015b). Berloff (2005) showed that upgradient eddy fluxes in a quasi-geostrophic model could be modelled statistically by fitting autoregressive processes to the eddy statistics obtained from an eddy resolving run, yielding an improved jet in a low-resolution model. Jansen and Held (2014) and Jansen et al. (2015) corrected spurious dissipation of kinetic energy by diagnosing the energy lost to gridscale viscosity and re-injecting this energy at larger spatial scales, so as to preserve energy conservation by mimicking the upscale cascade of energy that is expected in 2D turbulence. Porta Mana and Zanna (2014) designed an eddy parametrization by determining a function of the coarse-grained flow in a high-resolution quasi-geostrophic model that correlated well with the eddy forcing to serve as the basis for a stochastic parametrization that depends on the resolved scales. Such varied approaches all attempt to represent the rectified effects of upgradient momentum fluxes and energy backscatter (i.e. upscale energy transfer).

The approach of Porta Mana and Zanna (2014) was based on assuming that a more general stress-deformation relation than the standard eddy viscosity may apply to the flow of turbulent fluids. Rivlin and Ericksen (1955) showed how a fluid stress tensor can depend generally on the gradients of velocity as well as acceleration and so-called higher-order accelerations. The total stress tensor can be expressed as a summed series of tensors that depend on the strain tensor, with the strain tensor itself being the first term in the series. Hence truncation of the series retaining only its first term yields a diffusive forcing in the momentum and vorticity equations, and this type of fluid is termed a Newtonian fluid (Slemrod, 1999). Fluids for which further terms in the series contribute to the stress are termed non-Newtonian, and we assume here as Porta Mana and Zanna (2014) did that retaining these further terms in the series provides a way to model the turbulent stress. This approach to parametrizing turbulence is not new (Rivlin, 1957; Crow, 1968; Lumley, 1970; Meneveau and Katz, 2000); the novelty of our study is in the application of this approach to the quasi-geostrophic turbulence that characterises oceanic mesoscale eddies, specifically the eddy Reynolds stresses due to those eddies.

In assuming a non-Newtonian stress-strain relation to be valid for the turbulent flow, we are not assuming the ocean to be an actual non-Newtonian fluid (water is Newtonian). An example of an actual non-Newtonian flow is a fluid containing polymers, i.e. macro-molecules comprised of long chains of thousands of linked molecules (Spurk, 1997). Such fluids are known to show dependence of the viscosity on the fluid shear, analogously to how the (Smagorinsky, 1963) parametrization scheme represents turbulence with a viscosity coefficient that depends on the fluid deformation. The additional stress tensor terms for such fluids represent the rectified effects of the macro-molecule dynamics; for example, shear-thinning behaviour, where the fluid viscosity decreases at increased shear, can result from macro-molecules being more likely to get tangled together at lower shears. By attempting to model turbulence using similar mathematical expressions, we presume that turbulent eddies represent a "microstructure" in the fluid having some net effect on the "macro" flow that we wish to model. The heuristic justification for this approach is therefore similar to that of eddy viscosity (in which an analogy is made between random molecular motions and turbulent fluid motions). Just as laboratory experiments are used to validate stress-strain relations for actual non-Newtonian fluids, here we validate our approach using results from eddy-resolving numerical simulations of an ocean primitive equations model in an idealised configuration.

The paper is structured as follows. In Section 2 we consider possible stress tensors applicable to the ocean problem, and consider the effects of one proposed parametrization on momentum, vorticity, energy and enstrophy budgets. Section 3 then uses diagnostics from numerical simulations with a primitive equations model to evaluate the vorticity forcing by the proposed parametrization. In Section 4 we discuss how our approach relates the Reynolds stress tensor of the eddying flow. Conclusions are given in Section 5.

2. Theory

Our approach is to parametrize the horizontal eddy Reynolds stress tensor directly. The momentum and vorticity forcings due to parametrized eddies are then calculated from the stress tensor as flux divergences, and the parametrization satisfies conservation constraints if the relevant flux components vanish on the boundaries (e.g. Marshall et al., 2012). The parametrized Reynolds stress tensor will depend on the spatial gradients of velocity, and will be concisely expressed in terms of the flow deformation and the vorticity, allowing for an intuitive description of its behaviour. We will argue that the proposed stress tensor provides a parametrization that has the desirable quasi-geostrophic turbulence properties of allowing for upgradient fluxes and enstrophy dissipation.

The structure of the section is as follows. Section 2.1 introduces the deformation tensors on which the stress tensor may depend. Section 2.2 describes the momentum and vorticity forcing obtained from the proposed parametrization, and Section 2.3 considers its effects on energy and enstrophy. Section 2.4 then discusses its behaviour in more qualitative terms.

2.1. Form of the stress tensor

Consider a general stress tensor, $\boldsymbol{T}$, representing the effects of eddies on a two-dimensional (2D) fluid flow – that is, $\boldsymbol{T}$ is a parametrization for the horizontal eddy Reynolds stress tensor. The stress tensor divergence $\nabla \cdot \boldsymbol{T}$, where $\nabla \equiv (\partial_x, \partial_y)$, is a vector whose components give the forcing due to parametrized eddies in the corresponding vector components of the momentum equation: the x-component of $\nabla \cdot \boldsymbol{T}$ appears on the right-hand side (RHS) of the prognostic equation for the zonal velocity, u, and the y-component of $\nabla \cdot \boldsymbol{T}$ appears on the RHS of the equation for the meridional velocity, v. We assume an expression for $\boldsymbol{T}$ can be found that approximates the effects of small-scale turbulence on the large-scale flow. With respect to the assumption of 2D flow, al-