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Internal wave scattering in continental slope canyons, part 1: Theory and development of a ray tracing algorithm

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ABSTRACT

When internal waves interact with topography, such as continental slopes, they can transfer wave energy to local dissipation and diapycnal mixing. Submarine canyons comprise approximately ten percent of global continental slopes, and can enhance the local dissipation of internal wave energy, yet parameterizations of canyon mixing processes are currently missing from large-scale ocean models. As a first step in the development of such parameterizations, we conduct a parameter space study of M2 tidal-frequency, low-mode internal waves interacting with idealized V-shaped canyon topographies. Specifically, we examine the effects of varying the canyon mouth width, shape and slope of the thalweg (line of lowest elevation). This effort is divided into two parts. In the first part, presented here, we extend the theory of 3-dimensional internal wave reflection to a rotated coordinate system aligned with our idealized V-shaped canyons. Based on the updated linear internal wave reflection solution that we derive, we construct a ray tracing algorithm which traces a large number of rays (the discrete analog of a continuous wave) into the canyon region where they can scatter off topography. Although a ray tracing approach has been employed in other studies, we have, for the first time, used ray tracing to calculate changes in wavenumber and ray density which, in turn, can be used to calculate the Froude number (a measure of the likelihood of instability). We show that for canyons of intermediate aspect ratio, large spatial envelopes of instability can form in the presence of supercritical sidewalls. Additionally, the canyon height and length can modulate the Froude number. The second part of this study, a diagnosis of internal wave scattering in continental slope canyons using both numerical simulations and this ray tracing algorithm, as well as a test of robustness of the ray tracing, is presented in the companion article.

1. Introduction

Internal waves are efficient transmitters of energy across ocean basins. These waves, either generated by the winds or tidal flows over rough topography (Munk and Wunsch, 1998) propagate through the ocean basins until they are forced to break by topographic features, or non-linear wave-wave interactions (MacKinnon et al., 2013). To first approximation, 30–60% of the initial wave energy makes it away from the generation site and previous studies have shown that a sizable portion of this energy can make it to the edge of the basins where the continental slope is encountered (St. Laurent and Nash, 2004; Klymak et al., 2006; Alford et al., 2011; Waterhouse et al., 2014). At these continental slopes, internal waves are scattered, leading to higher wavenumbers and/or greater energy density. They can then become unstable (due to shear instability, convective instability or some combination thereof), break and lead to diapycnal mixing. Diapycnal mixing due to internal tide breaking, both near the generation site and in the farfield, is an important component of the global meridional overturning circulation (Ilicak and Vallis, 2012; Talley, 2013).

Over the past two decades, there have been numerous studies aimed at understanding the parameter space for which internal waves break over a host of farfield topographies, as well as at the generation site itself (Legg and Klymak, 2008; Nikurashin and Legg, 2011; Johnston et al., 2011). Plane slopes (Cacchione and Wunsch, 1974; Ivey and Nokes, 1989; Hallock and Field, 2005; Nash et al., 2007; Kunze et al., 2012; Hall et al., 2013), convex and concave slopes (Legg and Adcroft, 2003), ridges and mounts (Johnston and Merrifield, 2003; Klymak et al., 2013), channels (Drijfhout and Maas, 2007) and isolated/random topographic features (Egbert and

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Ray, 2000; Buehler and Holmes-Cerfon, 2011; Legg, 2014) have been studied from both observational and modeling perspectives.

Despite their potential to be a sink of internal tidal energy, continental slope canyons have been largely overlooked by the modeling community. As Kunze et al. (2002) suggested, submarine canyons may indeed be significant sinks of internal wave energy due to both their frequency along the continental slope and their geometries. Given that about 10% of the continental slope is carved out by such canyons, and that the geometry of the canyons may be conducive to wave focusing, Carter and Gregg (2002) argued from their observations of Monterey Canyon that the internal wave energy dissipation and subsequent mixing is non-negligible (approximately the same order of magnitude as the mixing currently parameterized in ocean general circulation models) and warrants study in more depth. This conclusion is in agreement with other observational studies of internal wave-driven mixing in continental slope canyons (Gordon and Marshall, 1976; Hotchkiss and Wunsch, 1982; Gardner, 1989; Petruncio et al., 1998; Codiga et al., 1999; Bosley et al., 2004; Bruno et al., 2006; Lee et al., 2009a; 2009b; Xu and Noble, 2009; Gregg et al., 2011; Hall and Carter, 2011; Waterhouse et al., 2013; Vlasenko et al., 2016).

While our study is motivated by observations of mixing in actual continental slope canyons, we begin with idealized V-shaped canyons in order to tease out the fundamental dynamics (we further justify this choice of topography in Section 4). We develop and employ a ray tracing algorithm to explore the impact of canyon geometry on ray focusing and wave number within a linear context. While ray tracing algorithms have been used to understand internal wave dynamics before (Manders and Maas, 2004; Maas, 2005; Drijfhout and Maas, 2007; Rabitti and Maas, 2013; 2014), for the first time, we use reflection information to calculate the Froude number (formally defined in Section 3) and hence estimate the likelihood for instability. Thus, without the use of a fully-nonlinear general circulation model (GCM), we aim to predict, using our linear ray tracing algorithm, where regions of instability may occur for wave scattering off idealized, V-shaped canyon topography.

The idealized canyons we have chosen to analyze are oversimplifications of real canyon bathymetry; however canyons tend to follow a roughly V-shaped profile (Shepard, 1981) and our focus here is not to capture every detail of particular wave-topography interaction, but to explore the parameter space (Carter and Gregg, 2002). Specifically, we do a suite of experiments to vary the geometric parameters of the ratio of canyon mouth opening to canyon length, as well as the shape/thalweg (line of lowest elevation) slope, to understand the wave reflection behavior and resulting instability. These are two important geometric parameters which vary between observed continental slope canyons, and thus a good starting point for our study (Gregg et al., 2011; Hall and Carter, 2011). As a further simplification, we only consider remotely-generated M2 tidal-frequency, mode-one internal waves, a reasonable assumption as a sizable fraction of the internal wave energy is observed to be at the M2 tidal-frequency (Munk and Wunsch, 1998).

The goal of Part 1 of this study is to extend the internal wave reflection theory to 3D, rotated topography and to use this theory to construct a ray tracing algorithm. Our objective in designing this ray tracing is to follow a large number of rays through the canyon region as they reflect off the topography, and store information on the trajectory of these rays. This stored information then allows us to (i) predict regions where instability is energetically possible and (ii) understand the processes that cause these regions to experience instability. In Part 2, through a comparison with numerical simulations, we also seek to test the robustness of this ray tracing derived from the linear theory. This ray tracing will then be used in tandem with a fully nonlinear GCM to understand the topographic control on wave breaking and subsequent energy loss (Nazarian and Legg, 2017).

In this paper, we build upon existing internal wave reflection theory and use this theory as the backbone of our ray tracing algorithm to understand internal wave scattering in continental slope canyons. In Section 2, we present the physical theory of 3D wave reflection. Based on this theory, we develop the methodology used in the linear ray tracing algorithm in Section 3 and present examples of the ray tracing for various idealized topographies. This ray tracing code may be applied to internal wave scattering off any arbitrary topography, as the algorithm depends only on the local topographic parameters. We then present the idealized canyons, and the justification for such canyons, in Section 4. In Section 5 we analyze the results of the ray tracing algorithm for two classes of idealized continental slope canyons to obtain a first-order understanding of internal wave dynamics in continental slope canyons. We find that the linear ray tracing algorithm predicts large envelopes of both increased ray density and, for canyons with non-vertical sidewalls, an increase in vertical wave number, both of which can contribute to the formation of instabilities. We also find that this region of instability due to topographic focusing is modified by the spatial extent of the canyon, including the canyon height and relative canyon length. In Part 2 of this study we will add to this understanding by complimenting the ray tracing with a fully-nonlinear GCM, as well as using the GCM to test the robustness of the ray tracing algorithm (Nazarian and Legg, 2017).

2. Theory

When low mode internal waves are scattered by topography, energy can be effectively transferred to higher vertical wavenumbers, which leads to a higher Froude number. This nondimensional number, Fr, quantifies the stability of the flow and the likelihood of transitioning into the turbulent flow regime. We thus develop the 3D internal wave scattering theory applied to a rotated coordinate system to calculate the wavenumber as a function of the geometry of the topography by which it is scattered, as well as the original wave properties. The original theory of internal wave reflection off topography was set forth by Phillips (1963; 1966) and considered further by Eriksen (1982). We adapt the setup of Eriksen (hereafter E82) to a rotated coordinate system to construct the two symmetric sides of the V-shaped canyon.

We start by considering a plane slope, inclined at an angle, α, relative to the horizontal. We then rotate the plane by another angle, ζ, relative to the y-axis. These angles are displayed in Fig. 1 and the resulting inclined, rotated plane comprises one sidewall of our V-shaped canyon.

Similar to E82, we consider a semi-infinite domain with x denoting the onshore direction, y denoting the alongshore direction and z denoting the vertical (as seen in Fig. 1). We start with the linearized, inviscid, non-rotating Boussinesq equations and assume that nondivergence is satisfied. Our guiding equations can thus be written as:

$$\left.\begin{aligned} &\frac{\partial u'}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \\ &\frac{\partial v'}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial y} \\ &N^2 w' + \frac{\partial^2 w'}{\partial t^2} = -\frac{1}{\rho_0}\frac{\partial^2 p'}{\partial t \partial z} \\ &\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \end{aligned}\right\} \tag{1}$$

where the prime notation denotes the wave perturbation fields. u, v and w are the velocity components in x, y and z, respectively, and p is the pressure. N^2 is the background density stratification, which is defined as $N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}$, where ρ_0 is the background density.