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Analytical approximations to spurious short-wave baroclinic instabilities in ocean models

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ABSTRACT

Most community ocean models that use z- or s-coordinates stagger their variables in the vertical using a Lorenz grid. Spurious short-wave baroclinic instabilities have been shown to occur on that grid by Arakawa and Moorthi. As the vertical resolution of the grid is improved, the wavelength of the spurious modes decreases and they become more and more trapped near one of the boundaries but they continue to grow at almost the same rate as the deep Eady/Charney modes. The spurious instabilities in the case of the Eady problem are here shown to be accurately reproduced by an analytical calculation which reduces the stability problem to a quadratic equation for their complex phase speeds. The interpretation of these spurious instabilities as resulting from spurious sheets of potential vorticity is revisited. A new interpretation is presented using a finite difference analogue of the Charney–Stern–Pedlosky integral constraint. This indicates that the spurious instabilities result from a vertical averaging of the advection of the relative vorticity which leads to a spurious interior source term in the finite difference potential vorticity equation.

1. Introduction

The vertical coordinate that an ocean model uses is generally recognised to be one of its primary design choices (Griffies et al., 2000). Accurate representation of the advection of tracers (Ilicak et al., 2012), the pressure gradients and the ocean bathymetry are all important considerations. Standard z coordinates typically represent the bathymetry as Lego-like blocks which is only first order accurate and forces the vertical velocity (directly below tracer points) at the model bottom to be zero. Terrain following s-coordinates have difficulties representing the horizontal pressure gradients sufficiently accurately over steeply sloping bathymetries (see for example references in Chu and Fan, 2003; Berntsen, 2011). Isopycnal (ρ) coordinates have poor resolution within the near surface mixed layers. Hybrids of z and ρ coordinates are generally considered to offer the most promising way forward.

The vertical staggering of variables is also an important model design choice. The main community ocean models that are written in (some form of) height or sigma coordinates, such as MOM5.0 (Griffies et al., 2012), NEMO (Madec, 2014) and ROMS (Shchepetkin and McWilliams, 2005) all use a Lorenz grid staggering. The original Lorenz grid staggering (Lorenz, 1960) for these vertical coordinates is depicted on the left-hand side of Fig. 1. The density, ρ, (used to calculate the buoyancy), tracers, T, pressure, p, and horizontal velocities, u and v, are stored at full levels. The vertical velocities are stored at half levels and the boundaries are at levels $1/2$ and $K+1/2$. The modified Lorenz grid (Arakawa, 1988), depicted on the right-hand side of Fig. 1, uses the same staggering except that the pressure is stored at the half-levels instead of at the full levels. If, when one uses the modified Lorenz grid, one calculates the horizontal pressure gradients at the full levels as simple averages of the pressure gradients at the half levels, the modified Lorenz grid is equivalent to the original Lorenz grid. Thus the original and modified Lorenz grids are often not distinguished and their properties are very similar (Arakawa, 1988). The formulations of Simmons and Burridge (1981), used by the ECMWF model for many years, and Arakawa and Suarez (1983) both use modified Lorenz grids (Bell, 2003).

The modified Lorenz grid is a very natural choice for a hydrostatic ocean model. Hydrostatic balance is calculated at full levels and the grid stretching is chosen so that this calculation is centred. The evolution of tracers and the incompressibility condition ($\nabla.\mathbf{u}=0$) are also calculated at full levels using velocities that are naturally centred at the cell faces when the C-grid formulation is chosen. The pressures are available at the corners of the velocity cells which makes calculation of the normal pressure forces on the

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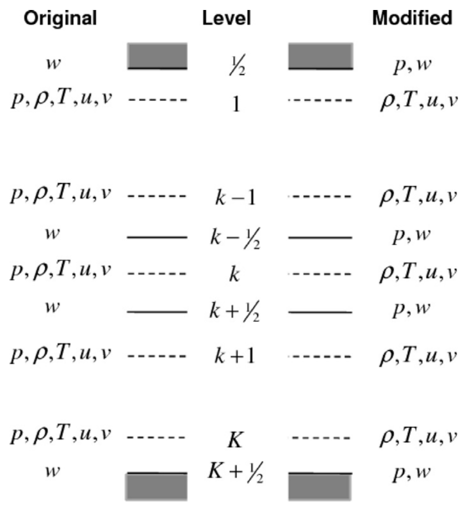

Fig. 1. The staggering of variables used by the original (left) and modified (right) Lorenz grids.

faces relatively easy. Finally the vertical velocities and the pressures are stored on the "horizontal" upper and lower boundaries. This is natural because the pressure at the sea surface is imposed by the atmosphere and the pressure on the bathymetry is explicitly calculated. Using Lorenz grids with flux formulations it is possible to conserve total energy, and the mean values and variance of tracers (Lorenz, 1960).

Although as just described they have many attractions, the Lorenz grids also have well known weaknesses. In particular the high vertical wavenumber normal modes are not optimal (Tokioka, 1978; Arakawa, 1988; Thuburn and Woollings, 2005). Vertical advection of density and tracers involves a two grid point vertical average of the vertical velocity which advects the background stratification. The horizontal pressure gradients are also two grid point vertical averages. This combination of averaging grossly reduces the equivalent depth of higher order vertical modes and allows a computational mode in which the density perturbation changes sign with height at each half grid level.

Furthermore Arakawa and Moorthi (1988) (hereafter AM) found that the Lorenz grid gives spurious short-wave baroclinic instabilities near the upper and lower boundaries when it is applied to the Eady problem. These normal mode instabilities, as they occur in the Eady problem for perturbations with no lateral variation, are illustrated by the blue crosses in Fig. 2. The growth rate[1] is displayed as a function of the wavelength of the instability on grids with (a) 6, (b) 20, (c) 40 and (d) 80 levels in the vertical. The modes in Fig. 2(a) with wavelengths greater than 250 km are the classical deep Eady/Charney modes and are accurately reproduced using just a few levels. There should be no unstable modes at shorter wavelengths. Unfortunately there are a number of spurious unstable modes at shorter wavelengths. The peak growth rate of the group of spurious waves with the shortest wavelengths is comparable with that of the Eady/Charney modes. As the vertical resolution of the grid increases, the wavelength of the spurious instabilities decreases but their peak growth rate does not diminish. AM found these instabilities first in quasi-geostrophic equations discretised using the Lorenz grid. They also found them in primitive equation models discretised using the Lorenz grid and showed that they have a significant detrimental impact on the surface fields.

The Charney–Phillips (C–P) grid (Charney and Phillips, 1953) uses an alternative vertical staggering of variables in which both the buoyancy and the vertical velocities are stored at the half-levels. This grid was originally introduced to solve the quasi-geostrophic equations and provides a more natural discretisation of them than the Lorenz grid. AM showed that the spurious instabilities obtained with the Lorenz grid are absent from the quasi-geostrophic Eady problem discretised using the C–P grid. Bell and White (1988)[2] showed that the N-level quasi-geostrophic Eady problem on the C–P grid can be solved analytically, giving a short-wave cut-off and no spurious short-wave instabilities.

Some atmospheric models (see e.g. Davies et al., 2005; Girard et al., 2014) have been re-engineered to use the C–P grid mainly for the above reasons. These models typically use semi-Lagrangian rather than flux-form advection schemes. Arakawa and Konor (1996) and Konor and Arakawa (1997) however devised schemes with good conservation properties for atmospheric models using the C–P grid and hybrid terrain following and isentropic coordinates. Their schemes have matching energy conversion terms in the thermodynamic and kinetic equations which sum to zero to conserve energy. They also use a flux form for the advection of temperature in the thermodynamic equation so that global mass integrals of functions of potential temperature are conserved under adiabatic processes. Although it appears that these schemes could be adapted for ocean models, the associated re-engineering of an ocean model would be a considerable undertaking. Accordingly this paper is devoted to an analysis of the spurious short-wave modes on the Lorenz grid and is intended to underpin an analysis of their likely occurrence and impact within ocean models. We note in passing that the simplest formulation of isopycnal ocean models uses a C–P grid (Bell et al., 2017) and that an investigation of the staggering used by hybrid coordinate models could be worthwhile.

Bell and White (1988)[3] analysed the spurious maxima and minima that occur in the (genuine) short-wave instabilities obtained for the Charney problem discretised using the C–P grid. They were able to simulate them quite well by assuming that the modes are trapped near one of the boundaries and that the potential vorticity gradient and details of the discretisation can be neglected except at the grid point closest to the critical layer. Here we apply these ideas to the discretization using the Lorenz grid and derive a quadratic equation for the phase speed of the spurious short-waves. The solutions of this quadratic are illustrated by the red plus signs in Fig. 2. It is evident that the quadratic equation reproduces the spurious short-wave modes very accurately (it does not of course represent the deep Eady/Charney modes at all well).

Section 2 summarises the quasi-geostrophic equations discretised using the modified Lorenz grid and the equations obtained by linearising them about a laterally uniform, vertically sheared, zonal flow. Section 3 explains the two approximations that we use to represent the spurious short-wave modes, derives a quadratic equation for their phase speeds, and examines it in some detail for the special case of the Eady problem. Section 4 discusses AM's interpretation of their results and suggests an improved interpretation based on a finite difference analogue of the Charney-Stern-Pedlosky integral constraints (Charney and Stern, 1962; Pedlosky, 1964). Section 5 summarises and presents a concluding discussion. The longer derivations are provided in appendices.

[1] Calculated using a Brunt–Vaisala frequency such that $N^2 = 10^{-4}\,\mathrm{s}^{-2}$, a water depth $H = 1000\,\mathrm{m}$, Coriolis parameter $f = 10^{-4}\,\mathrm{s}^{-1}$ and a zonal velocity difference between the top and bottom of $1\,\mathrm{ms}^{-1}$.

[2] The dispersion relation given by Bell and White (1988), their Eq (5), contains transcription errors: the factors $(1 + p^2/4\mathrm{N}^2)$ that multiply $\coth(\theta)$ and $\tanh(\theta)$ should both be square rooted; in these expressions N denotes the number of levels.

[3] In Eq. (20) of Bell and White (1988), c is misprinted as c_r.