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The viscous lee wave problem and its implications for ocean modelling

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ABSTRACT

Ocean circulation models employ 'turbulent' viscosity and diffusivity to represent unresolved subgridscale processes such as breaking internal waves. Computational power has now advanced sufficiently to permit regional ocean circulation models to be run at sufficiently high (100 m–1 km) horizontal resolution to resolve a significant part of the internal wave spectrum. Here we develop theory for boundary generated internal waves in such models, and in particular, where the waves dissipate their energy. We focus specifically on the steady lee wave problem where stationary waves are generated by a large-scale flow acting across ocean bottom topography. We generalise the energy flux expressions of [Bell, T., 1975. Topographically generated internal waves in the open ocean. J. Geophys. Res. 80, 320–327] to include the effect of arbitrary viscosity and diffusivity. Applying these results for realistic parameter choices we show that in the present generation of models with $\mathcal{O}(1)$ m^{2}s^{-1} horizontal viscosity/diffusivity boundary-generated waves will inevitably dissipate the majority of their energy within a few hundred metres of the boundary. This dissipation is a direct consequence of the artificially high viscosity/diffusivity, which is not always physically justified in numerical models. Hence, caution is necessary in comparing model results to ocean observations. Our theory further predicts that $\mathcal{O}(10^{-2})$ m^{2}s^{-1} horizontal and $\mathcal{O}(10^{-4})$ m^{2}s^{-1} vertical viscosity/diffusivity is required to achieve a qualitatively inviscid representation of internal wave dynamics in ocean models.

1. Introduction

Internal waves are an important mechanism for vertical and downscale transfer of energy in the ocean. Internal waves can transport energy from the upper and lower boundary of the ocean (where much of the energy is injected) to the ocean interior, where wave breaking and other nonlinear processes can lead to turbulent mixing (Waterhouse et al., 2014). Furthermore, they are amongst the larger scales of 'unbalanced' flow, and can therefore provide a conduit from large-scale 'balanced' flow to the small-scale turbulence where dissipation occurs (Vanneste, 2013). Internal waves are generated by surface wind stresses (Alford et al., 2016; Jouanno et al., 2016), tidal interactions with bathymetry (e.g. St Laurent and Garrett, 2002), geostrophic flows over rough topography on the sea floor (Nikurashin and Ferrari, 2010), and small-scale unbalanced flow at the ocean surface including submesoscale eddies, fronts and filaments (e.g. Danioux et al., 2012; Nagai et al., 2015; Shakespeare and Taylor, 2016).

Only recently have computational capabilities expanded sufficiently to permit regional ocean circulation models to be run at sufficiently high (100 m–1 km) horizontal resolution to resolve a significant portion of the internal wave spectrum (Nikurashin et al., 2013; Nagai et al., 2015; Rosso et al., 2015). In their 200 m resolution model, Nikurashin et al. (2013) find that the resolved waves generated via geostrophic flow over topography (lee waves) dissipate 80% of their energy in the water column directly above the topography. They extrapolate this result to the global ocean to suggest that the resolved waves with scales exceeding 1 km provide a first-order contribution to turbulent mixing directly above topography, thereby sustaining the ocean overturning circulation. Enhanced dissipation above rough topography is consistent with ocean observations (Waterhouse et al., 2014). However, observational estimates suggest that lee waves only dissipate 2–20% of their energy near topography (Waterman et al., 2013), much less than the 80% predicted from the Nikurashin et al. (2013) numerical model.

As with all large-scale ocean models, the subgrid-scale turbulence in wave-resolving numerical models must be parameterised, typically using Laplacian (or higher order) horizontal diffusivities

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and/or viscosities. The fact that the horizontal gridscale (100 m–1 km) is much larger than the vertical (1–20 m) implies that the corresponding viscosity/diffusivity will be that much larger: typical values of Laplacian horizontal diffusivities and/or viscosities employed in these high resolution models (e.g. Nikurashin et al., 2013; Nagai et al., 2015; Rosso et al., 2015) are of $\mathcal{O}(1)$ m^2 s^{-1} throughout the depth of the ocean. In comparison, values for vertical viscosity/diffusivity are typically of $\mathcal{O}(10^{-3}-10^{-5})$ m^2 s^{-1}. To some extent these parameterisations are intended to represent the effect of internal waves breaking and driving mixing of density and momentum in the ocean interior (Polzin, 2010; Polzin and Lvov, 2011). This situation presents a problem since we are parameterising the effect of waves while partially resolving waves, and thus any effect the parameterisation has on the waves is potentially a spurious one. Here we investigate this effect and what can be done to minimise or eliminate it. It it widely acknowledged that numerical ocean models should be run with the smallest possible turbulent viscosities and diffusivities[1], except in regions of the ocean where larger values can be physically justified. However, values considered 'small' change with model resolution as smaller-scale physics is explicitly represented— here we quantify what values are 'small enough' in the context of internal waves in wave-resolving models, and the constraint this places on model resolution.

The 'fluid' in the numerical models described above (which we will term the 'model fluid') has strongly non-isotropic behaviour, with order-of-magnitude different horizontal and vertical viscosity/diffusivity. As will be shown here, these parameter choices result in the energy loss from the wave field in large-scale numerical models often being dominated by the horizontal viscosity/diffusivity for much of the internal wave spectrum (excluding near-inertial waves). Most of these models also use the hydrostatic version of the Boussinesq equations, so we will only consider hydrostatic internal waves. Thus, our objective here is formulate hydrostatic linear internal wave theory to describe the energy flux associated with boundary-sourced internal waves in the presence of arbitrary viscosity/diffusivity. In particular, we will extend the classic steady lee wave energy flux expression of Bell (1975), which has been recently used to estimate lee wave generation in the global ocean (e.g. Nikurashin et al., 2014), to include viscous and diffusive effects. We describe this as the 'viscous lee wave problem'.

Viscous and diabatic internal waves have been investigated theoretically by previous authors, predominantly in the atmospheric context. Pitteway and Hines (1963) examined the decay of waves in upper atmosphere for isotropic viscosity and/or diffusivity. Yanowitch (1967) showed that finite viscosity can act to reflect upward-propagating waves in the atmosphere. These studies considered non-Boussinesq fluids and exponential density profiles appropriate to the atmosphere. Hazel (1967), following Booker and Bretherton (1967), showed that the reflection of waves at critical levels is unchanged by the presence of viscosity, using a similar approach to that taken in the present work of linearising about a background flow state. However, as noted above, here we focus specifically on ocean models with non-isotropic viscosity/diffusivity and apply our results to the lee wave problem.

2. Linear wave theory

Here we investigate the dynamics of internal waves generated at a boundary in a 'model fluid' with arbitrary viscosity and/or diffusivity using linear theory. The hydrostatic Boussinesq equations with uniform Laplacian diffusivity (horizontal κ_h; vertical κ_v) and viscosity (horizontal A_h; vertical A_v) on an f-plane, linearised about a state with spatially uniform and time-independent background flow, $\mathbf{U} = (U, V, 0)$, and spatially uniform and time-independent stratification, N^2, are

$$\frac{\partial u}{\partial t} - fv + \mathbf{U}\cdot\nabla_h u = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + A_h\nabla_h^2 u + A_v\frac{\partial^2 u}{\partial z^2}, \tag{1a}$$

$$\frac{\partial v}{\partial t} + fu + \mathbf{U}\cdot\nabla_h v = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + A_h\nabla_h^2 v + A_v\frac{\partial^2 v}{\partial z^2}, \tag{1b}$$

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + b, \tag{1c}$$

$$\frac{\partial b}{\partial t} + wN^2 + \mathbf{U}\cdot\nabla_h b = \kappa_h\nabla_h^2 b + \kappa_v\frac{\partial^2 b}{\partial z^2}, \tag{1d}$$

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}, \tag{1e}$$

where $(u,\ v,\ w)$ are the velocities in the $(x,\ y,\ z)$ Cartesian coordinate directions, p is the pressure, $b = -g(\rho-\rho_0)/\rho_0$ the buoyancy, f the Coriolis parameter, and ρ_0 the reference density. We seek solutions to (1) with the form of plane waves moving with the background flow,

$$\begin{aligned} b &= \widehat{b}(k,l,\omega,z)\,\exp\left(\imath(k(x-Ut)+l(y-Vt)+\omega t)\right) \\ &= \widehat{b}(k,l,\Omega,z)\,\exp\left(\imath(kx+ly+\Omega t)\right), \end{aligned} \tag{2}$$

where $\imath = \sqrt{-1}$, $k,\ l$ are the x and y wavenumbers, ω the Lagrangian frequency, and $\Omega = \omega - (kU + lV)$ the Doppler shifted (Eulerian) frequency. Our objective here is to determine the vertical structure function, $\widehat{b}(k,l,\omega,z)$, which describes the vertical amplitude profile of a wave, given the scale and frequency. Similar expressions to (2) apply for the velocity and pressure fields. Substituting these expressions into (1), and a little manipulation to eliminate the pressure, yields a system of equations,

$$\left(I\frac{\partial^2}{\partial z^2} + A\right)\cdot\mathbf{S} = \mathbf{0}, \tag{3a}$$

where I is the identity matrix,

$$\mathbf{S} = \begin{bmatrix} \widehat{\partial_z u_0} \\ \widehat{\partial_z v_0} \\ \widehat{w}_0 \\ \widehat{b}_0 \end{bmatrix} \tag{3b}$$

and

$$A = \begin{bmatrix} \imath\omega/A_v - K^2A_h/A_v & f/A_v & 0 & -\imath k/A_v \\ -f/A_v & \imath\omega/A_v - K^2A_h/A_v & 0 & -\imath l/A_v \\ \imath k & \imath l & 0 & 0 \\ 0 & 0 & -N^2/\kappa_v & \imath\omega/\kappa_v - K^2\kappa_h/\kappa_v \end{bmatrix} \tag{3c}$$

where $K^2 = k^2 + l^2$. Solutions to the system of Eq. (3) are given by

$$\mathbf{S} = \mathbf{S}(z=0)\,e^{\gamma z}, \quad (A+\gamma^2 I)\cdot\mathbf{S}(z=0) = 0, \tag{4}$$

for complex vertical wavenumber γ (eigenvalue γ^2). For non-trivial solutions we must have

$$\begin{aligned} 0 &= \det(A+\gamma^2 I) \\ &= \gamma^2(f^2 + (A_hK^2 - A_v\gamma^2 - \imath\omega)^2)(-K^2\kappa_h + \kappa_v\gamma^2 + \imath\omega) \\ &\quad + K^2\,N^2(A_hK^2 - A_v\gamma^2 - i\omega). \end{aligned} \tag{5}$$

[1] Here, 'smallest possible' refers to the value of explicit viscosity/diffusivity that ensures the flow field is smoothly represented on the model grid at any given location, and therefore depends on both resolution and flow properties. Values smaller than this lead to errors in the numerical advection scheme due to the poor representation of flow gradients (sometimes called 'numerical' diffusion/viscosity) and usually result in the model not being energetically consistent.