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Numerical modeling of submarine turbidity currents over erodible beds using unstructured grids

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ABSTRACT

Second-order central-upwind schemes proposed by Bryson et al. (2011) for the Saint–Venant system have two very attractive properties: well-balanced and positivity preserving, which are originally designed for constant fluid density and fixed beds in Bryson et al. (2011). For the turbidity current system with variable density over erodible beds, such desired properties can be obtained by developing a well-balanced and positivity preserving central-upwind scheme following the ideas in Bryson et al. (2011). To this end, in this paper, a coupled numerical model for two-dimensional depth-averaged turbidity current system over erodible beds is developed using finite volume method on triangular grids. The proposed numerical model is second-order accurate in space using piecewise linear reconstruction and third-order accurate in time using a strong stability preserving Runge–Kutta method. Applying the central-upwind method to estimate numerical fluxes through cell interfaces, the model can successfully deal with sharp gradients in turbidity flows. The developed numerical model can preserve the well-balanced property over irregular bottom, guarantee the non-negative turbidity current depth over erodible beds, and preserving the positivity of suspended sediment. These features of the developed numerical model and its robustness and accuracy are demonstrated in several numerical tests.

1. Introduction

Turbidity current is a type of underwater density current driven by the negative buoyancy forces due to the suspended sediment. Oceanic turbidity currents can be caused by tectonic disturbances of the sea floor, and the slumping of sediment that has piled up at the top of convergent plate margins, continental slopes and submarine canyons. Their profound impacts on the continental shelves, sea floor and ocean environment have attracted intensive research interests.

A large number of laboratory tests have been conducted to study the turbidity currents, see Parker (1982), Garcia et al. (1985), Parker et al. (1987), Bonnecaze et al. (1995), García (1993), Parker et al. (1986), Alexander and Mulder (2002), Baas et al. (2004), Cantelli et al. (2011), Hallworth and Huppert (1998), Sequeiros et al. (2009), Yu et al. (2006), Janocko et al. (2013), Motanated and Tice (2016), Chowdhury and Testik (2011) and Fedele and García (2009). However, laboratory studies are usually constrained by their small scale and can only be roughly applied to large-scale geophysical events. Only a few field observations of turbidity currents are reported in the literature due to the unpredictability and destructive nature of such events, see Inman et al. (1976), Hay (1987) and Normark (1989). Due to the aforementioned constrains of experimental and observational studies, numerical investigations based on theoretical and physical models have attracted significant attention because of their affordable computational cost and available details of time-dependent flow structure. Three-dimensional (3-D) models can resolve the vertical structure of turbidity current, particularly for the current front that may be considerably non-uniform in vertical. Several 3-D numerical investigation of density currents have been performed, see Huang et al. (2005), Kassem and Imran (2004), Necker et al. (2002), El-Gawad et al. (2012) and Ooi et al. (2015). However, the computational costs of such 3-D numerical models are very expensive, especially when applied to large scale cases. In addition, the physics of turbulent density currents has not been well understood; this also limits the applications of the 3-D turbidity current models. An alternative way to numerically investigate the submarine turbidity current is to use the depth-averaged two dimensional (2-D) numerical models which are attractive due to the fact that such 2-D models are well-understood, of lower computational cost and easy to use. One-layer 2-D depth averaged models have been widely studied. Parker et al. (1986) derived the depth-averaged governing equations for 2-D turbidity currents which provides the basic model for numerical

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studies. Choi (1998) developed a 2-D numerical model for turbidity currents using a dissipative-Galerkin finite element method. A deforming grid generation algorithm was used to track the current front. However, such techniques may become very expensive if the solution in the far-field is of interest because a large number of elements are required to produce accurate solutions. This prevents the extension of such a model to large-scale studies. Bradford and Katopodes (1999) developed a 2-D finite-volume method for turbid underflows considering non-uniform sediment. Roe scheme is used to estimate fluxes through cell interfaces. Their model didn't include the friction of ambient water which may lead to overestimation of propagating speed. Imran et al. (1998) developed a 2-D numerical model describing the formation of a submarine fan by a spreading turbidity current using finite difference method. In those models, the bed evolves in response to the exchange of suspended sediment in the turbidity current. However, the momentum and mass exchange along with sediment exchange are not considered in their studies. To fully consider the feedback impacts of bed deformation, Hu and Cao (2009) have modified the governing equations for turbidity current and extended it to 2-D cases (Hu et al., 2012a, 2012b) based on finite volume method. Those modified equations are also adopted in the current study.

One desired property of such numerical models for slender flows, especially for turbidity currents, is positivity preserving. Although applying CourantFriedrichsLewy (CFL) condition is sufficient to guarantee the numerical stability of finite volume methods, using the time step size satisfying CFL condition can not guarantee the non-negativity of current depth and volumetric concentration of sediment. None of the above-mentioned numerical models for turbidity currents incorporates the positivity preserving property. In applications, the current depth and volumetric concentration may be forced to be zero if the computed value is negative. This leads to a loss of certain amount of mass and momentum. In this study, the positivity preserving properties for both depth and concentration of turbidity current are guaranteed using the developed numerical model.

The developed numerical model in the current study is an extension of the well-balanced positivity preserving central-upwind methods developed by Bryson et al. (2011). However, the well-balanced and positivity preserving techniques in Bryson et al. (2011) were originally designed for Saint–Venant system with constant fluid density and fixed bed. Consequently, they can not be directly applied to the studied turbidity current system. To overcome this, the well-balanced and positivity techniques in Bryson et al. (2011) are modified in our proposed schemes, and a numerical model with such desired properties is developed.

In this paper, a numerical model based on the finite volume method, is developed to solve 2-D coupled depth-averaged turbidity current system consisting of sediment transport, hydrodynamic and morphodynamic models. The resulting hyperbolic system of balance laws consists of five coupled equations. The central-upwind method (Kurganov and Tadmor, 2000; Kurganov et al., 2001; Kurganov and Petrova, 2005, 2007; Kurganov and Levy, 2002) is adopted to estimate the numerical fluxes through cell interfaces. In order to guarantee the well-balanced property, special discretization for bed-slope source term is developed in this paper. Using non-negative reconstruction for bed, desingularization of point values and special treatment at wet-dry fronts, the model can deal with wetting and drying over complex geometry while obtaining the well balanced property at wet-dry boundaries. The positivity preserving property of the proposed model is proved.

It is noted that, due to the existence of ambient water, there is no true dry bed involved. The wetting-drying process mentioned in the current paper refers in particular to the wetting and drying process of turbidity current in the ambient water.

This paper is organized as follows. In Section 2, the governing equations and closures are presented, for which in Section 3 the well-balanced and positivity preserving techniques for turbidity current system are developed. Several numerical tests are conducted and presented in Section 4. Some concluding remarks complete the study in Section 5.

2. Governing equations

The governing equations for modeling the two-dimensional spreading of turbidity currents over erodible bed are conservation equations for fluid, sediment and bed material, respectively. A detailed derivation of the governing equations for 2-D turbidity currents can be found in Parker et al. (1986). In this study, the layer-averaged 2-D governing equations with non-equilibrium sediment transport adopted in Hu et al. (2012b) are used:

$$h_t + (hu)_x + (hv)_y = e_{\rm w}|V| + (E-D)/(1-p), \tag{1}$$

$$(hu)_t + \left(hu^2 + \frac{g'}{2}h^2\right)_x + (huv)_y = -g'hZ_x - (1+r_w)u_*^2 - \frac{u(E-D)(\rho_0-\rho)}{\rho(1-p)} - \frac{\rho_w-\rho}{\rho}ue_w|V|, \tag{2}$$

$$(hv)_t + (huv)_x + \left(hv^2 + \frac{g'}{2}h^2\right)_y = -g'hZ_y - (1+r_w)v_*^2 - \frac{v(E-D)(\rho_0-\rho)}{\rho(1-p)} - \frac{\rho_w-\rho}{\rho}ve_w|V|, \tag{3}$$

$$(hc)_t + (huc)_x + (hvc)_y = E - D. \tag{4}$$

The bed evolution is computed by using the Exner equation for the conservation of bed sediment, which takes the form:

$$Z_t = -\frac{E-D}{1-p}. \tag{5}$$

In Eqs. (1)–(5), t is time, x and y are horizontal coordinates, $h = h(x, y, t)$ is the thickness of the turbidity current layer, $u = u(x, y, t)$ and $v = v(x, y, t)$ are the x- and y-components of the layer-averaged velocities, respectively, $c = c(x, y, t)$ is the layer-averaged volumetric sediment concentration, $Z = Z(x, y, t)$ is the bed elevation, $|V| = \sqrt{u^2 + v^2}$ is the total velocity, g is the gravitational acceleration, $g' = Rgc$ is the submerged gravitational acceleration, $R = (\rho_s - \rho_w)/\rho$ is submerged specific gravity, ρ_w and ρ_s are the densities of water and sediment, respectively, $\rho = \rho_w(1 - c) + \rho_s c$ is the density of the water-sediment mixture, p is the bed porosity; $\rho_0 = \rho_w p + \rho_s(1 - p)$ is the density of the saturated bed, $u_* = \sqrt{C_D u|V|}$ and $v_* = \sqrt{C_D v|V|}$ are the components of the friction source in the x- and y-directions, respectively, C_D is the bed drag coefficient with a typical range 0.002-0.06 (Parker et al., 1987), r_w is the ratio of upper-interface resistance to bed resistance with $r_w = 0.43$ (Parker et al., 1986), e_w is a fluid entrainment coefficient which is estimated by Parker et al. (1986):

$$e_w = \frac{0.00153}{0.0204 + \mathrm{Ri}}, \tag{6}$$

where Ri is Richardson number defined by:

$$\mathrm{Ri} = \frac{Rghc}{\sqrt{u^2+v^2}}. \tag{7}$$

Furthermore, in (1)–(5), $E = \omega E_s$ is the sediment entrainment, $D = \omega c_b$ is the sediment deposition, ω is the settling velocity of sediment calculated as (Zhang and Xie, 1993)

$$\omega = \sqrt{(13.95\nu/d)^2 + 1.09(\rho_s/\rho_w - 1)gd} - 13.95\nu/d, \tag{8}$$