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Evaluation of a scalar eddy transport coefficient based on geometric constraints

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ABSTRACT

A suite of idealized models is used to evaluate and compare several previously proposed scalings for the eddy transport coefficient in downgradient mesoscale eddy closures. Of special interest in this comparison is a scaling introduced as part of the eddy parameterization framework of Marshall et al. (2012), which is derived using the inherent geometry of the Eliassen–Palm eddy flux tensor. The primary advantage of using this coefficient in a downgradient closure is that all dimensional terms are explicitly specified and the only uncertainty is a nondimensional parameter, α, which is bounded by one in magnitude.

In each model a set of passive tracers is initialized, whose flux statistics are used to invert for the eddy-induced tracer transport. Unlike previous work, where this technique has been employed to diagnose the tensor coefficient of a linear flux-gradient relationship, the idealization of these models allows the lateral eddy transport to be described by a scalar coefficient. The skill of the extant scalings is then measured by comparing their predicted values against the coefficients diagnosed using this method. The Marshall et al. (2012), scaling is shown to scale most closely with the diagnosed coefficients across all simulations. It is shown that the skill of this scaling is due to its functional dependence on the total eddy energy, and that this scaling provides an excellent match to the diagnosed fluxes even in the limit of constant α. Possible extensions to this work, including how to incorporate the resultant transport coefficient into the Gent and McWilliams parameterization, are discussed.

1. Introduction

The development of ocean eddy parameterizations continues to be an area of vigorous research. The ubiquity of geostrophic ocean eddies, and the central role they play in shaping the mean circulation, stratification, and transport of tracers of the ocean (e.g. Danabasoglu et al., 1994; Henning and Vallis, 2004; Marshall and Speer, 2012; Lauderdale et al., 2013), implies that the skill of eddy parameterizations can have a significant effect on the accuracy of future climate predictions. Furthermore, it is likely that parameterizations will be necessary even for the largest-scale ocean eddies well into the foreseeable future. To resolve the geostrophic eddy field and accurately represent the complex interactions between these eddies and the large-scale circulation requires model grid spacings at least an order of magnitude finer than the dominant energy-containing scales. Even in the mid-latitudes, where the dominant eddy scale is approximately 100 km (Stammer, 1997; Chelton et al., 1998), for a model to be considered “mesoscale eddy-resolving” requires a grid spacing of less than 10 km (Hecht and Smith, 2008; Hallberg, 2013), beyond the capability of current climate-scale ocean models.

A longstanding approach to the eddy parameterization problem is to consider the resolved flow as an averaged or filtered representation of the true flow field. For a Cartesian-coordinate model, after applying the standard Reynolds averaging axioms to the primitive equations the resulting equation set contains an eddy flux divergence in each of the constituent equations, each of which must be parameterized. It has heretofore been common to develop parameterizations for each eddy flux individually, rather than developing a single, unified parameterization for the full set of eddy fluxes. The downside of this approach is that a model may feature several potentially inconsistent eddy parameterizations, where answers to practical questions such as how these parameterizations interact are often unknown.

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Because of these difficulties, it is advantageous to try to reduce the number of required parameterizations by grouping the eddy forcing into as few equations as possible. The residual-mean formalism (e.g. Andrews and McIntyre, 1976; Andrews, 1983; de Szoeke and Bennett, 1993; McDougall and McIntosh, 2001; Young, 2012; Maddison and Marshall, 2013) is one means by which this can be achieved through careful averaging and the appropriate definition of a residual circulation. In addition to their mathematical elegance, the residual-mean equations have shown promise as a platform for ocean model development in scenarios where knowledge of the Eulerian velocity is not necessary (e.g. Wardle and Marshall, 2000; Ferreira and Marshall, 2006; Zhao and Vallis, 2008).

With regard to the eddy parameterization problem, it has been shown by Marshall et al. (2012) that the quasigeostrophic (QG) residual-mean formalism can be used to develop a framework for eddy parameterization which conserves momentum and satisfies important energy constraints. A subtle yet important feature of this framework is that the problem of understanding, quantifying, and parameterizing eddy-mean flow interaction can be effectively recast as a problem of understanding the underlying geometry of the eddy fluxes themselves. The Eliassen–Palm flux tensor (hereafter "EP tensor"), which is introduced in Section 2.1 and described in detail in Maddison and Marshall (2013), is a fundamental object describing this geometry, and among its noteworthy features is that it can be chosen such that the resultant eddy stresses are nonzero only in the horizontal momentum equations. From a practical point of view this offers significant advantages for the development of eddy parameterizations, allowing a modeler to avoid imposing separate (and possibly physically inconsistent) parameterizations in the momentum and buoyancy equations.

As of the writing of this paper, no single eddy closure has been developed which skillfully parameterizes each of the terms in the EP tensor in a unified and consistent manner. Many of the most common eddy parameterizations instead rely on the phenomenology of turbulence at a particular scale to parameterize specific components of the tensor. For example, the popular Gent and McWilliams scheme (Gent and McWilliams, 1990; Gent et al., 1995, hereinafter GM) is a parameterization for the eddy tracer fluxes induced by mesoscale baroclinic turbulence, effectively closing only for the vertical fluxes appearing in the bottom row of the EP tensor and only in the limit of large-scale, along-isopycnal flow. The GM parameterization holds particular appeal because it can both be thought of in residual-mean context as introducing an "eddy transport velocity" (offering potential advantages for the numerical implementation of the scheme, e.g. Griffies, 1998; Griffies et al., 1998), and also through its relationship to other downgradient diffusive closures (e.g. Redi, 1982). The latter point has prompted ocean modelers to explore the relationship between the transport coefficients of the GM and Redi parameterizations (e.g. Dukowicz and Smith, 1997; Griffies, 1998; Abernathey et al., 2013; Bachman and Fox-Kemper, 2013), and to develop techniques to ensure that these parameterizations are scale-aware (e.g. Bachman et al., 2016; Pearson et al., 2016) and satisfy appropriate boundary conditions (e.g. Aiki et al., 2004; Ferrari et al., 2008; 2010).

It is now widely recognized that the GM and Redi transport coefficient must vary both spatially and temporally, though optimal choices for these coefficients remains an open question. Many proposed choices have appeared in the years since the GM and Redi parameterizations were initially developed (Redi, 1982; Gent and McWilliams, 1990; Gent et al., 1995) and concatenated (Griffies, 1998) for practical use. The values of the proposed coefficients have been informed by a variety of methods, including baroclinic instability theory (Visbeck et al., 1997; Killworth, 1997), adjoint modeling (Ferreira et al., 2005), energetic arguments (Cessi, 2008; Eden and Greatbatch, 2008; Marshall and Adcroft, 2010), parcel excursion theory (Fox-Kemper et al., 2008), and direct diagnosis (Bachman and Fox-Kemper, 2013). While each of these proposals has shown promise in replicating key eddy transport characteristics in specific model configurations, their skill at matching diagnosed buoyancy diffusivities has never been compared in a systematic way. In this paper such a systematic comparison will be performed using a suite of idealized models. As the GM parameterization was designed to mimic the restratification and available potential energy extraction of mesoscale baroclinic instability, the basic test case for this comparison will be the spindown of a baroclinically unstable front (e.g. Bachman and Fox-Kemper, 2013).

Included among the list of coefficients in this comparison is an expression for the GM transport coefficient that is inferred using the geometric framework of Marshall et al. (2012). It will be shown that this expression exhibits greater skill at matching the diagnosed buoyancy diffusivities at all times during the baroclinic spindown across the full range of model initial conditions. The goal of this paper will be to highlight the skill of this closure, and in doing so to demonstrate a practical use for the geometric framework and its nontraditional approach to the eddy parameterization problem. This is intended as a possible first step towards a more unified treatment of parameterizing subgridscale eddy fluxes, wherein all terms comprising the EP tensor would be represented in a physically consistent way that conserves energy and momentum.

The outline of this paper is as follows. In Section 2.1 the geometric framework will be reviewed and it will be shown how this leads to a prescription for the GM transport coefficient. The basic theory of downgradient, mesoscale eddy closures is reviewed in Section 2.2, along with the extant scalings for the transport coefficient that are compared using the modeling suite. Section 3 discusses the numerical models used to test the skill of these scalings and presents the results from the comparison. A discussion of the implications of these results, along with concluding remarks, appears in Section 4.

2. Background and theory

2.1. Using the geometric framework to infer an eddy transport coefficient

The "eddy" component of a flow variable is typically defined as the deviation away from some average, and additional advantages are gained when the averaging operation is defined so as to reduce the complexity of the resulting equations of motion. Of particular interest are averaging operations which permit the equations of motion to be rewritten in residual-mean form (e.g. Andrews and McIntyre, 1976; Andrews, 1983; de Szoeke and Bennett, 1993; McDougall and McIntosh, 2001; Young, 2012; Maddison and Marshall, 2013).

Residual-mean theory has previously been used in conjunction with the QG approximation to yield various forms of an eddy flux tensor whose double divergence describes the time tendency (hereafter "eddy tendency") of QG potential vorticity (e.g. Hoskins et al., 1983; Plumb, 1986; Cronin, 1996) due to turbulence. More recently, this approach has been extended to the hydrostatic Boussinesq primitive equations (Young, 2012; Maddison and Marshall, 2013), where the associated eddy flux tensor still provides information on the eddy tendency of (Ertel) potential vorticity, but appears in the momentum equations instead of the potential vorticity conservation equation. If it is assumed that the buoyancy increases strictly monotonically with height, the resulting equation set can be written in Cartesian coordinates as

$$\frac{D^{\#}\hat{\mathbf{u}}}{Dt} + f\hat{\mathbf{k}} \times \hat{\mathbf{u}} + \nabla_h p^{\#} = \mathcal{F} - \nabla_3 \cdot \mathbf{E}, \quad (1)$$