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Approximation of wave action flux velocity in strongly sheared mean flows

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ABSTRACT

Spectral wave models based on the wave action equation typically use a theoretical framework based on depth uniform current to account for current effects on waves. In the real world, however, currents often have variations over depth. Several recent studies have made use of a depth-weighted current $\tilde{U}$ due to [Skop, R. A., 1987. Approximate dispersion relation for wave-current interactions. J. Waterway, Port, Coastal, and Ocean Eng. 113, 187–195.] or [Kirby, J. T., Chen, T., 1989. Surface waves on vertically sheared flows: approximate dispersion relations. J. Geophys. Res. 94, 1013–1027.] in order to account for the effect of vertical current shear. Use of the depth-weighted velocity, which is a function of wavenumber (or frequency and direction) has been further simplified in recent applications by only utilizing a weighted current based on the spectral peak wavenumber. These applications do not typically take into account the dependence of $\tilde{U}$ on wave number k, as well as erroneously identifying $\tilde{U}$ as the proper choice for current velocity in the wave action equation. Here, we derive a corrected expression for the current component of the group velocity. We demonstrate its consistency using analytic results for a current with constant vorticity, and numerical results for a measured, strongly-sheared current profile obtained in the Columbia River. The effect of choosing a single value for current velocity based on the peak wave frequency is examined, and we suggest an alternate strategy, involving a Taylor series expansion about the peak frequency, which should significantly extend the range of accuracy of current estimates available to the wave model with minimal additional programming and data transfer.

1. Introduction

Important theoretical advances have been made in the last several decades which have advanced our understanding of wave-current interaction in ocean circulation. Theories have been incorporated in numerical models with the main intent of including wind wave effects in ocean circulation without resolving surface gravity wave motions for computational efficiency. Within typical modeling systems, an ocean circulation model is coupled with a wave generation and propagation model in order to determine wave effects on currents and vice versa. The spectral wave models include the effect of the mean flow in the computation of wave action flux, and the ocean circulation models account for the wave-averaged wave forcing driving or modifying the mean flow.

Spectral wave models are usually based on the theory for waves in the presence of depth-uniform currents. In the real world, however, currents are usually vertically sheared to some degree. Recently, various studies (van der Westhuysen and Lesser, 2007; Arduin et al., 2008; Warner et al., 2010) have suggested the use of a depth-weighted current $\tilde{U}(k)$ as the basis for the wave-current interaction in propagation models, where $\tilde{U}(k)$ is the first order correction to the phase speed for an arbitrarily varying current $U(z)$ and is given by

$$\tilde{U}(k) = \frac{2k}{\sinh 2kh} \int_{-h}^{0} U(z) \cosh 2k(h+z)\, dz \tag{1}$$

where h is the water depth and k is the wave number (Skop, 1987; Kirby and Chen, 1989). In application, this approach is often further truncated by using $\tilde{U}(k_p)$ as the representative value of $\tilde{U}$ for all wave components, where k_p denotes the wavenumber at the spectral peak frequency. This procedure is now included as an option in widely used models such as Delft-3D and COAWST (Elias et al., 2012; Kumar et al., 2011; 2012). We remark here that the perturbation scheme of Kirby and Chen (1989), defined originally for the case of weak current, can be straightforwardly modified to cover the case of a strong current with weak additional shear. Assuming a fairly arbitrary split between a depth uniform and depth varying current

$$U(z) = U_0 + \alpha U_1(z); \qquad \alpha \ll 1 \tag{2}$$

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and repeating the procedure used to develop the solution in Kirby and Chen quickly establishes that the choice for leading order current speed is $U_0 = \tilde{U}$, with the details of the overall solution maintained up to second order. The parameter α represents the magnitude of current shear; a scaling analysis based on finite depth waves with horizontal and vertical length scales proportional to k^{-1}, leads naturally to an expression

$$\alpha = \frac{\Omega}{kU_s} \tag{3}$$

where Ω characterizes the maximum value of shear in the current profile, and U_s is the surface current speed. The expressions developed in both the perturbation solution and the analytic solution for constant shear discussed below are both easier to interpret using a slightly different expression

$$\alpha = \frac{\Omega h}{U_s} \tag{4}$$

which is used throughout the remainder of the paper.

The purpose of this study is to demonstrate the inappropriateness of the use of the weighted current $\tilde{U}$ as the current component of the group velocity, and to examine the effect of using either the correct or incorrect estimate of the current speed evaluated only at the spectral peak frequency. We evaluate the accuracy of approximate solutions in comparison to analytical or numerical solutions for the full theory based on the Rayleigh stability equation. The theory described here is limited to unidirectional propagation on a following or opposing current, and so currents and wave numbers appear as scalars rather than vectors. In Section 2, the problem for a linear wave in a uniform domain with arbitrary current $U(z)$ is established. We then outline the common approximations for group velocity used in modeling and the errors resulting in these applications. In Section 3, we evaluate the approximations for the analytic case of a wave on a current with constant vorticity, and establish the consistency of the expressions for group velocity derived from the perturbation solution of Kirby and Chen (1989). Section 4 examines comparable results of the numerical solution for a current profile measured at the mouth of the Columbia River (MCR) (Kilcher and Nash, 2010). In Section 5, we evaluate the shortcomings of practical approximations in existing coupled circulation-spectral wave models, where it is typical to use only $\tilde{U}(k_p)$ as the current speed. Finally, in Section 6 we describe a strategy for providing a compact but significantly more accurate representation of current advection velocity in SWAN or similar models, using a Taylor series expansion of the expression for the wavenumber-dependent current speed about the reference value at the peak frequency.

2. Theory and approximate expressions for the absolute group velocity C_{ga}

2.1. General theory

We consider the linearized wave motion of an incompressible, inviscid fluid, with wave number $\mathbf{k}$ and phase velocity $\mathbf{C_a} = \omega\mathbf{k}/k^2$, propagating on a stream of velocity $\mathbf{U}(z)$ in finite water depth h. Current and depth variables are assumed to be uniform in horizontal directions (Fig. 1). ω denotes the absolute wave frequency in a stationary frame of reference, which also fixes the value of $\mathbf{U}(z)$. We seek solutions for the vertical component of the wave orbital velocity

$$w(\mathbf{x}, z, t) = \tilde{w}(z)e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{5}$$

The problem for the vertical structure of plane waves in a spatially uniform domain, riding on a vertically sheared current $\mathbf{U}(z)$, is then given by an extension of the Rayleigh equation to allow for

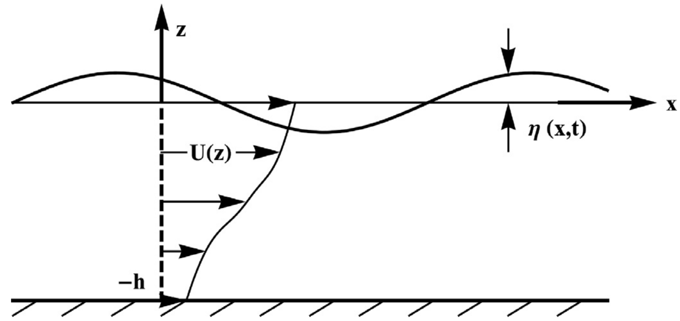

Fig. 1. Definition sketch.

an oblique angle between wave and current direction as well as possible rotation of the current vector over depth

$$\begin{aligned} \sigma(z)(\tilde{w}'' - k^2\tilde{w}) - \sigma''(z)\tilde{w} &= 0; && -h \le z \le 0 \\ \sigma_s^2\tilde{w}' - [gk^2 + \sigma_s\sigma']\tilde{w} &= 0; && z = 0 \\ \tilde{w} &= 0; && z = -h \end{aligned} \tag{6}$$

where primes denote differentiation with respect to z and g is the gravitational constant. The quantity $\sigma(z) = \omega - \mathbf{k}\cdot\mathbf{U}(z)$ represents a depth-varying relative frequency, with σ_s denoting the value at the mean surface $z = 0$. The separate use of the kinematic surface boundary condition for a surface wave of form $\eta = a\exp i(\mathbf{k}\cdot\mathbf{x} - \omega\mathbf{t})$ gives $\tilde{w}(0) = -i\sigma_s a$.

The model (6) has been used in a number of studies of arbitrary or idealized velocity distributions; see reviews by Peregrine (1976), Jonsson (1990) and Thomas and Klopman (1997). For the general case of arbitrary $U(z)$, Voronovich (1976) has described the conservation law, in the geometric optics approximation, for an adiabatic invariant corresponding to the wave action density. Evaluation of these results requires knowledge of a solution to (6), however. Karageorgis (2012) has shown a method for constructing expressions for the dispersion relation for waves on a number of vertical vorticity distributions, but does not consider the further determination of the group velocity.

For the case of weak shear, solutions to (6) may be obtained using a perturbation approach, described to leading order for deep water by Stewart and Joy (1974) and extended to finite depth by Skop (1987) and to second order by Kirby and Chen (1989). Considering deep water waves, Shrira (1993) has further demonstrated how series solutions may be extended to high order. Alternately, numerical solutions may be obtained using a shooting method due to Fenton (1973). In the following, we limit ourselves to the evaluation of the first and second-order solutions presented in Kirby and Chen (1989) and further limit ourselves to waves and currents propagating in the same direction. For definiteness, we suppose that waves are propagating towards the right with $c > 0$ and $k > 0$, while the current can be propagating in either $\pm\, x$ direction.

2.2. Perturbation solution of Kirby and Chen (1989)

Following Kirby and Chen (1989), we assume that the steady current velocity is small relative to some measure of wave phase speed. Here, we use a Froude number based on the surface velocity $U_s = U(0)$ defined by

$$F = \frac{U_s}{\sqrt{gh}}; \qquad |F| \ll 1 \tag{7}$$

The wave phase speed is given by

$$C_a = \frac{\omega}{k} = C_0 + (F)C_1 + (F^2)C_2 + O(F^3) \tag{8}$$

where we indicate ordering w/r F schematically and retain dimensional expressions for now. C_0 is the usual result for linear waves