Ocean Modelling 116 (2017) 146–158

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Modelling and observations of oceanic nonlinear internal wave packets affected by the Earth's rotation

Roger Grimshaw[a,*], Jose C.B. da Silva[b], Jorge M. Magalhaes[b]

[a] Department of Mathematics, University College London, London, U.K.
[b] Department of Geosciences, Environment, and Spatial Planning, Interdisciplinary Centre of Marine and Environmental Research University of Porto, Porto, Portugal

ARTICLE INFO

Article history:
Received 25 February 2017
Revised 23 June 2017
Accepted 25 June 2017
Available online 27 June 2017

Keywords:
Internal solitary waves
Rotation
SAR images

ABSTRACT

The large-amplitude internal solitary waves commonly observed in the coastal ocean can propagate for long distances for long times, so that it may be necessary to take account of the effects of the Earth's background rotation. In this case an appropriate model wave evolution equation is the Ostrovsky equation, whose typical solutions indicate that internal solitary waves will evolve into envelope wave packets. Unlike the more usual Korteweg-de Vries solutions which are typically rank-ordered wave packets, these are centred with the largest waves in the middle. This qualitative feature, together with certain key quantitative parameters such as the envelope carrier wavenumber and speed, can be sought in oceanic observations. Hence we have examined many SAR images of internal solitary waves with the general aim of finding features indicating that rotational effects have become significant. From these we report in detail on six typical cases of which four give indications of rotational effects. In addition we use a two-layer fluid model to estimate how the rotational parameters depend on the background stratification and topography.

1. Internal solitary waves in a rotating reference frame: the Ostrovsky equation

Large-amplitude internal solitary waves are commonly observed to propagate in the coastal ocean, see the reviews by Grimshaw (2001), Holloway et al. (2001), Ostrovsky and Stepanyants (2005), Helfrich and Melville (2006), Grimshaw et al. (2007, 2010) and the book by Vlasenko et al. (2005). It is now widely accepted that the basic paradigm for these waves is based on the Korteweg-de Vries (KdV) equation, first derived in this context by Benney (1966) and Benjamin (1966) and subsequently by many others, see the aforementioned references. In a reference frame moving with a linear long wave speed c, the KdV equation is

$$\eta_t + \mu\eta\eta_x + \lambda\eta_{xxx} = 0. \quad (1)$$

Here the subscripts denote partial derivatives, $\eta(x, t)$ is the amplitude of the linear long wave mode $\phi(z)$ corresponding to a linear long wave with phase speed c, which is determined from the modal equation,

$$\{\rho_0(c-u_0)^2\phi_z\}_z - g\rho_{0z}\phi = 0, \quad \text{for} \quad -h < z < 0, \quad (2)$$

$$\text{and} \quad \phi = 0 \quad \text{at} \quad z = -h, \quad (c-u_0)^2\phi_z = g\phi, \quad \text{at} \quad z = 0. \quad (3)$$

Here $\rho_0(z)$ is the stably stratified background density stratification, and $u_0(z)$ is a horizontal background shear current. The coefficients μ and λ are given by

$$I\mu = 3\int_{-h}^{0} \rho_0(c-u_0)^2\phi_z^3\, dz, \quad (4)$$

$$I\lambda = \int_{-h}^{0} \rho_0(c-u_0)^2\phi^2\, dz, \quad (5)$$

$$I = 2\int_{-h}^{0} \rho_0(c-u_0)\phi_z^2\, dz. \quad (6)$$

In general the modal Eqs. (2) and (3) support an infinite number of modes, but here as is customary in the literature, we will confine attention to the first baroclinic mode.

However, these oceanic internal waves are often observed to propagate for long distances over several inertial periods, and hence the effect of the Earth's background rotation is potentially significant. A prominent example are the large ISWs that propagate across the South China Sea, see Zhao and Alford (2006) and Alford et al. (2010). There are also numerous remote sensing images throughout the coastal oceans that show multiple wave packets, see Jackson (2004), separated by the M_2 period, indicating that

* Corresponding author.
E-mail address: r.grimshaw@ucl.ac.uk (R. Grimshaw).

http://dx.doi.org/10.1016/j.ocemod.2017.06.009
1463-5003/

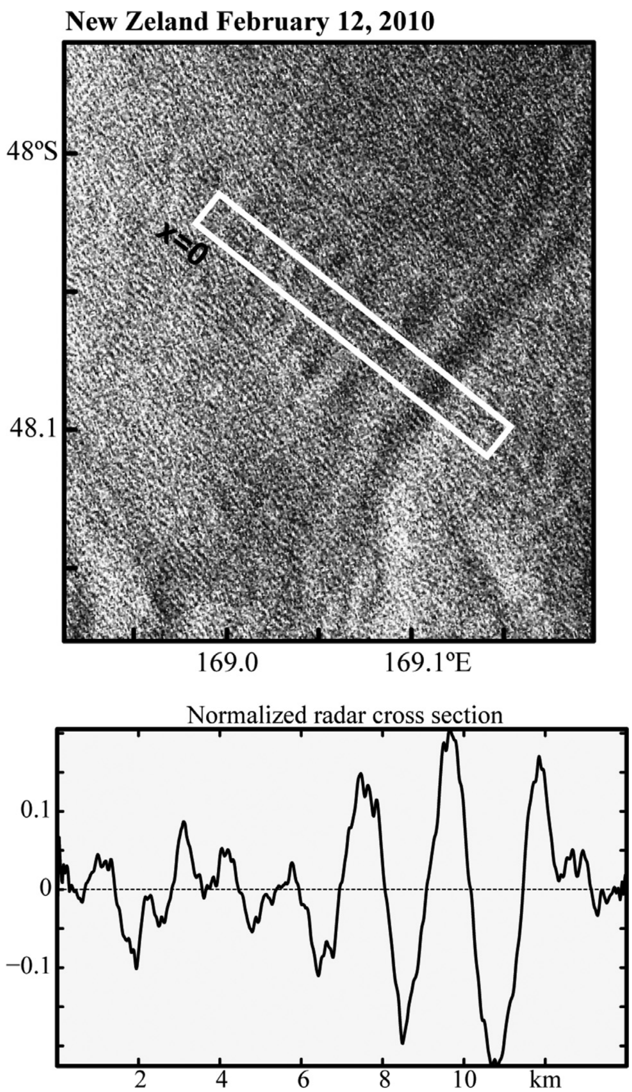

Fig. 1. SAR image and backscatter profile for the New Zealand Plateau.

the ISWs persist over periods longer than the local inertial period. The relevant extension of the KdV Eq. (1) that includes the effects of rotation is the Ostrovsky equation, derived initially by Ostrovsky (1978), and later in a two-dimensional framework by Grimshaw (1985),

$$\{\eta_t + \mu\eta\eta_x + \lambda\eta_{xxx}\}_x = \gamma\eta. \tag{7}$$

The background rotation is represented by the coefficient γ given by, Grimshaw (2013),

$$I\gamma = f^2\int_{-h}^{0}\rho_0\Phi\phi_z\,dz, \quad \rho_0(c-u_0)\Phi = \rho_0(c-u_0)\phi_z - (\rho_0u_0)_z\phi. \tag{8}$$

where f is the Coriolis parameter. In the absence of a background current $(u_0 = 0)$ $\Phi = \phi_z$ and so $\gamma = f^2/2c$. In this case, the usual situation in the ocean, from (5) and (8) we see that $\lambda\gamma > 0$, and then it is known that Eq. (7) does not support a steady solitary wave solution, see Grimshaw and Helfrich (2012) and the references therein. The simplest explanation is that then the additional term on the right-hand side of (7) removes the spectral gap in which solitary waves exist for the KdV equation, and hence no solitary waves are expected to occur. Note that when there is a non-zero background current, then it is possible, but very unlikely in oceanic conditions, that $\lambda\gamma < 0$, Grimshaw (2013). Nevertheless, if this should occur, then the Ostrovsky Eq. (7) does support a solitary wave, albeit of envelope type, see Grimshaw et al. (1998, 2016) and Obregon and Stepanyants (1998).

Next, we review briefly the salient features of the Ostrovsky Eq. (7) needed to interpret SAR observations. For the normal case when $\lambda\gamma > 0$ Grimshaw and Helfrich (2008), Grimshaw and Helfrich (2012) and Grimshaw et al. (2013) have shown that the long-time effect of rotation is the destruction of an initial ISW by the radiation of small-amplitude inertia-gravity waves, and the eventual emergence of a coherent steadily propagating nonlinear envelope wave packet typical of nonlinear Schrodinger (NLS) models. Analogous behaviour was found by Helfrich (2007) and Helfrich and Grimshaw (2008) in numerical simulations of fully nonlinear models of a two-layer fluid. The packet envelope propagates with a speed close to the maximum group velocity c_m. For the Ostrovsky Eq. (7) the linear dispersion relation for sinusoidal waves of wavenumber k, frequency ω is

$$c_p = \frac{\omega}{k} = \frac{\gamma}{k^2} - \lambda k^2, \tag{9}$$

where c_p is the phase velocity. The corresponding group velocity is

$$c_g = \frac{d\omega}{dk} = -\frac{\gamma}{k^2} - 3\lambda k^2. \tag{10}$$

Since we can assume without loss of generality that $c > 0$, it follows that then $\lambda > 0$, $\gamma > 0$, c_g is negative for all wavenumbers k, and has a local maximum where $dc_g/dk = 0$ at $k = k_m$ where $3\lambda k_m^4 = \gamma$; the maximum in the group velocity is $c_m = -2\gamma/k_m^2 = -2\sqrt{3\gamma\lambda}$. Note that as γ increases so does k_m, $|c_m|$.

The numerical simulations reported by Grimshaw and Helfrich (2008), Grimshaw and Helfrich (2012) and Grimshaw et al. (2013) show that an initial KdV solitary wave decays, emitting radiating inertia-gravity waves and is extinguished in a finite time, followed by the emergence of a nonlinear wave packet, with envelope speed close to c_m and a carrier wavenumber close to k_m. Importantly for our later comparisons with observations we note that the weakly nonlinear theory of Grimshaw and Helfrich (2008) predicts that the carrier wavenumber is enhanced by nonlinearity and becomes $k_c = 3k_m/2$. Once formed, this packet persists and remains coherent for a very long time. Suppose that the initial condition $\eta(x, 0) = \eta_0(x)$ is the KdV solitary wave with amplitude a_0,

$$\eta_0(x) = a_o\,\mathrm{sech}^2(K_0x), \quad \mu a_0 = 12\lambda K_0^2. \tag{11}$$

At a later time the amplitude a_s is given by, see the aforementioned references,

$$|a_s|^{1/2} = |a_0|^{1/2} - \Gamma t, \quad \Gamma = \gamma\sqrt{\frac{12\lambda}{|\mu|}}. \tag{12}$$

Thus the solitary wave is extinguished in a finite time

$$t_e = \frac{1}{\gamma}\left\{\frac{|\mu a_0|}{12\lambda}\right\}^{1/2}, \tag{13}$$

which is proportional to $|a_0|^{1/2}/\gamma$. Since more general initial conditions for the KdV equation will produce a train of solitary waves, we conjecture that again, under the influence of rotation, these will be extinguished and replaced with envelope wave packets.

Next, by examining the integrability or otherwise of the reduced Ostrovsky equation, that is (7) with the third-order linear dispersive term omitted, Grimshaw et al. (2012) showed that rotation inhibits nonlinear steepening, and hence the formation of solitary-like waves, consistent with results found numerically by Gerkema and Zimmerman (1995), Gerkema (1996) and Helfrich (2007). Grimshaw et al. (2012) defined the Ostrovsky number,