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# A mathematical investigation of the air-ground temperature relationship in permafrost regions on the Tibetan Plateau

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## ABSTRACT

Air and soil temperatures are important factors that contribute to hydro-thermal processes and ecosystem dynamics in permafrost regions. However, there is little research regarding soil thermal dynamics during freeze-thaw processes in permafrost regions with thermal orbits on the Tibetan Plateau. Thermal orbits can provide simplified illustrations of the relationships between air and ground temperatures. This paper presents a new quantitative analysis for thermal orbits by combining the characteristics of ellipse and linear regression theories. A sensibility analysis of thermal orbits was conducted with different air and ground temperatures and vegetation types on the Tibetan Plateau. Results indicated that the thermal orbit regression slopes and intercepts had variations in characteristics between air and ground temperatures at different depths. More specifically, both air and ground temperatures showed homologous variation with increasing depth. This type of analysis is important for a better understanding of permafrost thermal properties as they relate to soil moisture, climate change, and vegetation effects in permafrost regions on the Tibetan Plateau.

## 1. Introduction

Air and ground temperatures are key near-surface variables that influence land surface processes, climate change and biogeochemical cycles in the atmosphere–water–vegetation–land continuum (Cristóbal et al., 2008; Holmes et al., 2008; Mostovoy et al., 2006; Nagler et al., 2005). Previous reports have studied the air-ground temperature relationships (Geiger et al., 2009; Goodrich, 1982; Hinkel and Outcalt, 1993). The process governing the energy transfer between ground and atmosphere is complex, influenced not only by air temperature variations, but also by vegetation, snow cover, and soil moisture variations. Latent heat exchanges and solar radiation changes at or near the ground surface can also affect such energy transfer (Beltrami and Kellman, 2003; Geiger, 1965). Meanwhile, ground temperatures drive and modulate the response of soil biogeochemical processes to changes in air temperature such as $CO_2$ production and surface flux emissions (Beltrami and Kellman, 2003; Lloyd and Taylor, 1994; Luo et al., 2001; Risk et al., 2002). The interaction between all these variables over different time scales results in a complex relationship between air and ground temperature (Beltrami, 1996; Goodrich, 1982; Hinkel and Outcalt, 1993; Paul et al., 2004). More recently, studies have sought to analyze additional factors that might influence this relationship, including carbon cycle and climate warming (Smerdon et al., 2009). Ground temperature controls ecological processes and influences plant growing and shallow ground temperature has a close relationship with air temperature (Körner and Paulsen, 2004). Moreover, soil respiration has also been shown to be sensitive to changes of ground temperature (Bond-Lamberty and Thomson, 2010). Considering the important roles air and ground temperatures play in various processes, it is necessary to better explore their relationship to improve our understanding of climate warming and related heat processes.

In prior research, approximations of both the descriptions and assumptions regarding the interplay between air and ground temperatures have proven helpful in understanding their relationship (Carslaw and Jaeger, 1959; Demetrescu et al., 2007; Smerdon et al., 2004, 2006). Variations in air temperature can be considered to follow a sinusoidal oscillation over a one-year period. Similarly, if heat is transferred by conduction, ground temperature generally follows a sinusoidal oscillation as well (Geiger et al., 2009; Smith and Riseborough, 1983). The conductive signals penetrating the subsurface have been well-described (Carslaw and Jaeger, 1959; Carson, 1963; Geiger et al., 2009). For example, Beltrami (1996) introduced a phase-space representation of air and ground temperatures, termed *thermal orbits*, which provided a semi-quantitative description of the coupling between air and ground

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temperatures. Thermal orbits are generated by plotting daily or monthly ground temperature versus an air temperature time series. This plot yields elliptical phase-space orbits with characteristics that depend on the amplitude attenuation and phase shift of the propagating subsurface signal that are relative to the air temperature signal. This is analogous to the well-known Lissajous figures found in electronic circuit analysis (Al-Khazali and Askari, 2012; Beltrami, 1996; Smerdon et al., 2009). To date, several lines of work have shown that the concept of a thermal orbit is helpful to understanding the relationship between air and ground temperatures (Beltrami, 1996, 2001; Sushama et al., 2007). Beltrami (1996) distinguished between conductive or non-conductive heat in the permafrost using thermal orbit, while Sushama et al. (2007) simulated permafrost changes for the 21st century in North America by combing a regional climate with thermal orbits. However, the utility of thermal orbits did not allow for a quantitative analysis of subsurface temperature variations at different depths (Smerdon et al., 2009). A quantitative method was used by Smerdon et al. (2009) to simulate the relationship between air and ground temperatures by plotting ground temperatures versus air temperatures, resulting in data that traced an ellipse. Additional work defined the slope of all lines tangent to the ellipse using the calculus of parameterized curves. Results allowed for the description of the thermal orbit changes in both time and space. However, this study mainly focused on ellipses under ideal conditions, where vertical tangents occurred (Smerdon et al., 2009). Results from the prior thermal orbit study were approximate and did not consider changes in the intercept for thermal orbits. Therefore, it is important to further improve upon this idealized method to allow for better analysis of thermal orbits changes in comparison with observational data.

Changes in the climate and thermal properties at the soil surface result in active layer thermal dynamic changes and subsequent permafrost changes. Therefore, the active layer has been regarded as a changeable heat resistance between the air and ground temperatures in the permafrost (Lachenbruch, 1994). Given this, changes in the permafrost would result in changes to active layer properties. The active layer was also a significant key factor in climate change and hydrological systems (Burn and Smith, 1988; Goodrich, 1978; Hinzman et al., 1991; Kane et al., 1991; Lachenbruch, 1994; Romanovsky and Osterkamp, 1995). Climate warming has resulted in increased winter air temperatures and considerable increases to soil surface temperatures in summer on the Tibetan Plateau (Zhao et al., 2004). Critically, this has resulted in permafrost degradation in this region. Several studies have evaluated the thermal dynamics of the active layer on the Tibetan Plateau, with a focus on hydrothermal processes (Hu et al., 2014; Zhao et al., 2011), soil thermodynamic parameters (Li et al., 2014), soil organic content (Wang et al., 2014) and modeling hydrothermal processes (Hu et al., 2014, 2015a, 2015b). Past work has also shown that changes to the active layer soil thermal regime have impacts on soil development processes (Darmody et al., 2004; Thorn et al., 2002) and carbon cycle on a global scale (Field and Raupach, 2004). Given this, it is necessary to recognize regional variations in active layer thermal properties in the permafrost region (Zhang et al., 2003) and collect more detailed annual data related to the active layer thermal regime on the Tibetan Plateau using thermal orbits. As such, the aims of this study were three-fold: (1) To present a new quantitative method that considers thermal orbit regression slope and intercept; (2) To evaluate the performance of this new method in permafrost regions; and (3) To analyze trends between air-ground temperatures and different types of vegetation on the Tibetan Plateau.

## 2. Site and data description

The air temperature was taken from a height of 1.5 m and soil temperature and moisture at different soil depths were monitored at four sites (Fig. 1) in different eco-regions. Temperature was measured using thermostats with a resolution of ± 0.5 °C and soil moisture was measured using a Hydra soil moisture sensor (Frequency Domain Reflectometer) with an accuracy of ± 2.5% (Hu et al., 2015a). The data from these four different eco-regions in 2012 were used here. The main geographical characteristics of the four sites are listed in Table 1.

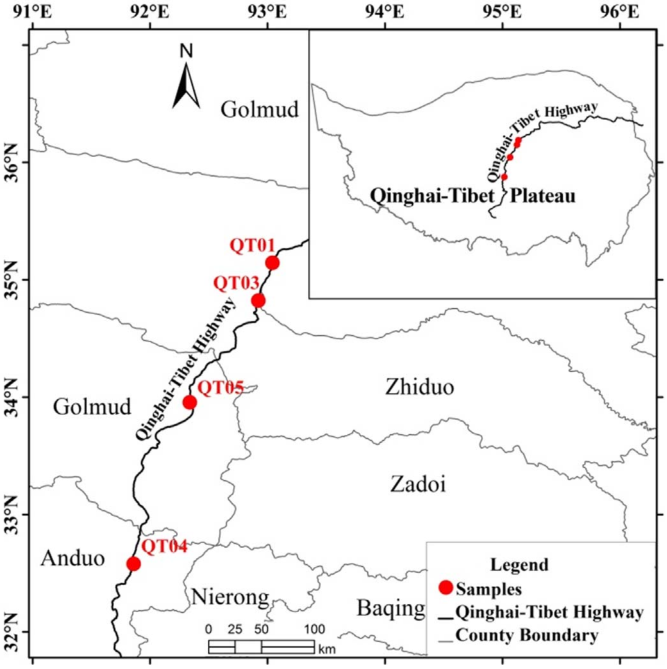

**Fig. 1.** Map of monitoring sites on the Tibetan Plateau.

## 3. Methods

### 3.1. Thermal conduction theory

Based on thermal conduction theory, the surface air temperature (SAT) change was shown to be in line with the sinusoidal function given by the following (Beltrami, 1996):

$$T(0,t) = \overline{T}_0 + A\sin(\omega t + \varphi) \tag{1}$$

With the surface temperature boundary condition given in Eq. (1), the ground temperature (z) was calculated as follows:

$$T(z,t) = \overline{T} + A\exp(-\alpha z)\sin(\omega t + \varphi - \alpha z) \tag{2}$$

where, $\overline{T}_0$ and $\overline{T}$ were the mean SAT and ground temperature, and $A$ and $\varphi$ were the surface air temperature initial amplitude and phase, respectively. $\alpha = \sqrt{\omega/2k}$ and k were the thermal diffusivity (Carslaw and Jaeger, 1959).

### 3.2. Development of thermal orbit regression method

The following equations show the parameterization process taken to quantitatively describe the thermal orbit characteristics between air and ground temperatures. Eqs. (1) and (2) were parameterized as follows:

$$x(t) = \overline{T}_0 + A\sin(\omega t + \varphi) \tag{3}$$

and

$$y(t) = \overline{T} + A\exp(-\alpha z)\sin(\omega t + \varphi - \alpha z) \tag{4}$$

The combination of Eqs. (3) and (4) give:

$$\begin{cases} x\text{-}\overline{T}_0 = A\sin(\omega t + \varphi) \\ y\text{-}\overline{T} = A\exp(\text{-}\alpha z)\sin(\omega t + \varphi - \alpha z) \end{cases} \tag{5}$$

The parameterized equations gave rise to an elliptical pattern when