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Approximate solution of a one-dimensional soil water infiltration equation based on the Brooks-Corey model

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ABSTRACT

The solution of a one-dimensional unsaturated soil water infiltration equation can provide theoretical support when simulating soil water distribution. To solve the equation for one-dimensional vertical infiltration of soil moisture in unsaturated soil with initially uniform moisture content distribution, a new method is proposed based on the Principle of Least Action and the Variational Principle. We used second-, third- and fourth-order Taylor series expansions to obtain an approximate analytical solution for a one-dimensional, constant water head, vertical infiltration equation. The results were compared with a numerical solution obtained using the HYDRUS-1D software. Our results showed that the soil water content profile, accumulated infiltration and infiltration rate can be calculated by the new method, and the soil water content calculated by the second-, third- and fourth-order approximate analytical solutions was closer to the numerical solution in the higher water content areas. The relative error of the soil water content for clay loam, silt, loam and sandy loam ranged from 0.7187% to 3.0848%, 0.9667% to 2.2847% 0.6310% to 2.2943%, 1.7504% to 6.8823%, respectively. In four soils modeled numerically, the fourth-order results were the best, with the smallest relative errors and the best coefficients of determination. The longer the wetting front distance, the smaller the calculated errors. Thus, the fourth-order approximate analytical solution can be used to simulate the long-term vertical infiltration process. If the results do not require the highest error precision, the second-order approximate analytical solution was simple in form and also sufficient to be used to describe the water content of the vertical infiltration process. Our results show that the approximate analytical solution proposed provides an effective approach to predict soil water content in the vertical infiltration process.

1. Introduction

Soil water infiltration (Bruce and Klute, 1956; Gardner, 1958; Philip, 1957; Zhang et al., 2010; Parlange, 1972) plays a pivotal role in the hydrological cycle. It is particularly important in a wide range of aspects of engineering, such as hydrological runoff yield calculation, groundwater resources evaluation, farmland irrigation, farmland drainage and prevention of soil salinization, all of which have practical applications. The soil water content, cumulative infiltration, infiltration rate and wetting front distance are important indices allowing a determination to be made of the accuracy of simulation results with respect to unsaturated soil water movement. It is, therefore, necessary to have an understanding of the functional relationships between these variables in order to analyze the features of soil water movement. Due to the complexity of the Richards' equation, which describes water movement, and the limitations of the technique to solve it, the exact solution for the Richards' equation has still not been fully elucidated. In order to simulate the process of unsaturated soil water movement and derive the soil water characteristic curve, in-depth research has been undertaken in both China and elsewhere (Parlange et al., 1997; Wang et al., 2002, 2003, 2004; Ma et al., 2009, 2010, 2015; Basha, 2011; Hayek, 2014, 2015a, 2015b, 2016). Although there has been a great effort to improve analytical methods, all of those currently employed are based on hypothetical approaches.

The various analytical and numerical methods originally proposed by Bruce and Klute (1956), Green et al. (1986), Klute and Dirksen (1986), Broadbridge and White (1988) and Srivastava and Yeh (1991), Chen et al. (2001, 2003) are, conceptually, relatively clear but they are time consuming and expensive. The horizontal absorption method, however, is very promising because it is simple; for example, Wang et al. (2000) presented a theoretical analysis of horizontal one dimensional unsteady soil water transfer. Infiltration equations have been verified by experimental data and the results indicate that the equations are reasonable. Zlotnik et al. (2007)

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presented an approach referred to as the "launch pad" technique, which is based on the traveling wave solution, to generate an exact solution of the boundary value problem for the Richards' equation. Ma et al. (2009, 2010) obtained a simple approximation solution to the horizontal infiltration problem with a low initial moisture content and initially uniform soil moisture distribution by utilizing the exponential distribution, and they established that there was a relationship between the cumulative infiltration and wetting front distance based on the Brooks-Corey model parameters. In order to explore the surge irrigation effect rule on the water distribution in the aeration zone with shallow groundwater, Fu et al. (2015) studied the water distribution and the infiltration effect law by examining a combination of intermittent infiltration and indoor water infiltration using a model simulation. The laboratory test was intended to allow them to observe and analyze data relating to soil moisture, cumulative infiltration and wetting front depth associated with irrigation with shallow groundwater during surge flow irrigation. Zhang et al. (2016) developed a finite analytical method that combines the flexibility of numerical approaches and the advantages of analytical solutions based on head-based and mixed-form Richards' equations.

However, vertical infiltration plays a pivotal role in the hydrological cycle. A new method for estimating soil hydraulic parameters in the Brooks-Corey model has been proposed by Ma et al. (2011). This method, investigated using one-dimensional vertical infiltration experiments, showed that the estimates of the Brooks-Corey soil parameters were quite comparable to those derived using the horizontal absorption method when the gravitational potential is balanced by the matric potential (capillary potential) and the soil stays wet for a sufficiently long time. Using the estimated parameters, the infiltration process simulated by the Brooks-Corey model is in good agreement with the observations. An analytical solution for one-dimensional steady vertical flux through unsaturated homogeneous soils has been presented by Hayek (2015a, 2015b). The model assumes power law hydraulic conductivity, diffusivity functions and that the finite-depth flow medium overlying a water table is the soil domain. A steady constant flux is applied at the top boundary while a constant saturation value is specified at the bottom boundary. The algebraic solution can be inverted back to obtain exact explicit solutions when the power law parameters are related. The analytical solution can be used for comparing models, validating numerical solutions, as well as for estimating the hydraulic parameters. Therefore, research examining the analytical solution of the unsaturated soil vertical infiltration problem has a certain practical significance. Wang et al. (2003) proposed an algebraic model of one-dimensional transient soil water infiltration with three parameters, and were able to derive the soil moisture content, cumulative infiltration, infiltration rate, water flux and distance of the wetting front advance. A parameter estimation method for the Brooks-Corey model parameter is given by Wang et al. (2003). Ma et al. (2015) derived an approximate analytical solution similar to Green-Ampt model for one-dimensional vertical infiltration into soils with initially uniform soil moisture distribution. However, all of the methods discussed are based on a number of assumptions.

Principle of Least Action (Bork and Zellweger, 1969; Provost, 1975; Taylor, 2003; Gray and Taylor, 2007) is a method used to explain the law of objective things in physics. It was historically called "least" because its solution requires finding the path that has the least value. Thus, this principle can be used to finding the optimal path to describe the soil water movement in the unsaturated soil. The main objectives of this study, therefore, were to 1) develop the approximate analytical solutions of the one-dimensional vertical infiltration equation entirely based on the Principle of Least Action and the Variational Principle; 2) compare the analytical solution with the numerical solution obtained using the HYDRUS-1D software Šimůnek et al. (2012, 2013); 3) discuss and compare the feasibility of analytical solutions proposed by Wang et al. (2003), Ma et al. (2015) and this paper.

2. Materials and methods

2.1. Methods and theory

The Buckingham-Darcy law equation describing one-dimensional vertical water movement (infiltration) in unsaturated soil can be described with the following equation:

$$q(h) = K(h)\frac{dh}{dz} + K(h) \tag{1}$$

where $q(h)$ is soil water flux density ($cm \cdot min^{-1}$), $K(h)$ is unsaturated hydraulic conductivity ($cm \cdot min^{-1}$), h is the water pressure head (cm), and z is the vertical position (cm), with downward as the positive direction. Based on the Buckingham-Darcy law, the Richards' equation for zero-ponded water infiltration into a one-dimensional vertical column of homogeneous, uniformly unsaturated soil can be described as:

$$\begin{cases} \dfrac{\partial\theta}{\partial t} = \dfrac{\partial q}{\partial z} = \dfrac{\partial}{\partial z}\left[K(h)\dfrac{\partial h}{\partial z}\right] + \dfrac{\partial K(h)}{\partial z} \\ \theta(z,0) = \theta_0 \\ \theta(0,t) = \theta_s \\ \theta(\infty,t) = \theta_0 \end{cases} \tag{2}$$

where, θ is the soil water content ($cm^3 \cdot min^{-3}$), θ_s is the saturated soil water content ($cm^3 \cdot min^{-3}$), θ_0 is the initial or 'backgound' soil water content ($cm^3 \cdot min^{-3}$).

Brooks and Corey (1964) proposed a model to describe the soil water retention curve as follows:

$$S_e = \begin{cases} \dfrac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\dfrac{h_d}{h}\right)^n, & h \le h_d \\ 1, & h > h_d \end{cases} \tag{3}$$

where, θ_r is the residual water content ($cm^3 \cdot min^{-3}$), S_e is effective saturation, h_d is bubbling pressure (cm), and n is an index parameter representing the soil's pore size distribution.

The unsaturated hydraulic conductivity $K(h)$ in Eq. (1) can be expressed as (Brooks and Corey, 1964):

$$K(h) = K_s\left(\frac{h_d}{h}\right)^m = K_s\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{\frac{m}{n}} \tag{4}$$

where K_s is saturated hydraulic conductivity ($cm \cdot min^{-1}$), m is a parameter, and $m = 3n + 2$.

Since Newton's time, classical mechanics has been elegantly reformulated as a single unifying principle known as Hamilton's principle. Following Feynman et al. (1964) and Landau and Lifshitz (1976), Hamilton's principle is frequently called "the principle of least action." According to the least action formulation of classical mechanics, a particle moves along the path for which the action is a minimum. In this paper, the principle of least action (Taylor, 2003; Gray and Taylor, 2007) was used to analyze the one-dimensional vertical infiltration problem (Eq. (2)), and the solution of this problem is obtained by using the variational principle. The principle of least action is to find an extremum (maximum or minimum value) for a problem that is a functional extreme value problem. In practice, to solve the extreme value of the function the variational method must be used. The principle of least action is to find the solution that results in the least amount of action in all possible circumstances, so the one-dimensional vertical infiltration problem can be described as finding a curve $\theta(z)$ satisfying the Brooks-Corey model which determines the soil water content changes the fastest from θ_s to θ_0 than the other curves.

Based on the principle of least action, the Richards' equation in Eq. (2) can be converted into another expression as follows:

$$\frac{dz}{dt} = \frac{dq}{d\theta} \tag{5}$$