Geoderma 287 (2017) 54–65

Contents lists available at ScienceDirect

Geoderma

journal homepage: www.elsevier.com/locate/geoderma

Multifractal analysis in soil properties: Spatial signal versus mass distribution

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ARTICLE INFO

Article history:
Received 22 January 2016
Received in revised form 26 July 2016
Accepted 4 August 2016
Available online 5 September 2016

Keywords:
Mass function
Multifractal spectrum
Generalized Structure Function
Scaling exponent function

ABSTRACT

The spatial variability of soil properties is a constant expected factor that must be considered in soil studies. This variability is composed of "functional" variations and random fluctuations or noise. Multifractal formalism is suitable for variables with self-similar distributions on a spatial domain. Multifractal analysis can provide insight into the spatial variability of soil parameters. In soil science, it has been quite popular to characterize the scaling property of a variable measured along a transect as a mass distribution of a statistical measure on a length domain of the studied transect. The analysed variable is divided into a number of self-similar segments, and the partition function and mass function are estimated. Based on these estimations, the multifractal spectrum (MFS) is calculated. Another technique that can be applied focuses on the variations of a measure by analysing the absolute differences in the soil property values at different scales, such as the Generalized Structure Function (GSF) and the Universal Multifractal Model (UMM). The aim of this study was to compare both types of multifractal methods on a set of soil physical properties measured on a common 1024 m transect across arable fields at Silsoe in Bedfordshire, East-Central England. The studied properties were total porosity (Porosity), gravimetric water content (GWC) and nitrous oxide flux (N_2O flux). The results showed that when using both methods, the N_2O flux exhibits a distinctive multifractal character, and weak multifractal characters are detected in the GWC and Porosity cases. Additionally, several parameters were calculated and discussed.
Finally, the relationship between the mass exponent function ($\tau(q)$) and the GSF ($\zeta(q)$) found in the literature, was positively verified for the three variables. On the contrary, the relationship between $\zeta(q)$ and the scaling exponent function based on UMM (K(q)) showed discrepancies in N_2O flux and GWC for q values higher than 3. This is the first time that these comparisons have been made on soil property data.

1. Introduction

Soil properties, such as pH, soil moisture, and porosity, vary spatially and exhibit strong fluctuations even over short distances. This variability is due to the combined action of physical, chemical and biological processes that operate with different intensities and at different scales. The description and quantification of the spatial variability of soil properties are important for modelling soil processes (Burrough et al., 1994).

This variability is composed of "functional" (defined) variations and random fluctuations or noise (Goovaerts, 1997, 1998). However, the distinction between these two components is scale dependent because increasing the scale of observation almost always reveals structure in the noise (Logsdon et al., 2008). Geostatistical methods and, more recently, multifractal/wavelet techniques have been used to characterize the scaling and heterogeneity of soil properties along with other methods originating from complexity science (de Bartolo et al., 2011).

Many fractal/multifractal methods have been developed to characterize these features over the years. Halsey et al. (1986) formulated the fixed-size box-counting algorithm, which is the most common classical multifractal analysis (MFA) method, to calculate the multifractal exponents, such as the scaling exponent ($\tau(q)$) and the generalized fractal dimension D(q). This method has been widely used in many soil science studies (Folorunso et al., 1994; Kravchenko et al., 2002, 2003; Vereecken et al., 2007).

Hurst (1951) proposed a rescaled range analysis (R/S analysis) to study the Nile and the problems related to water storage. More importantly, he proposed an important exponent, generally known as the Hurst exponent, to quantify the long-range correlations of the signal series. However, the R/S analysis can only handle stationary signals. To handle the fluctuations in non-stationary signals, new methods arise mainly from cascade models and turbulence studies (Davis et

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http://dx.doi.org/10.1016/j.geoderma.2016.08.004
0016-7061/

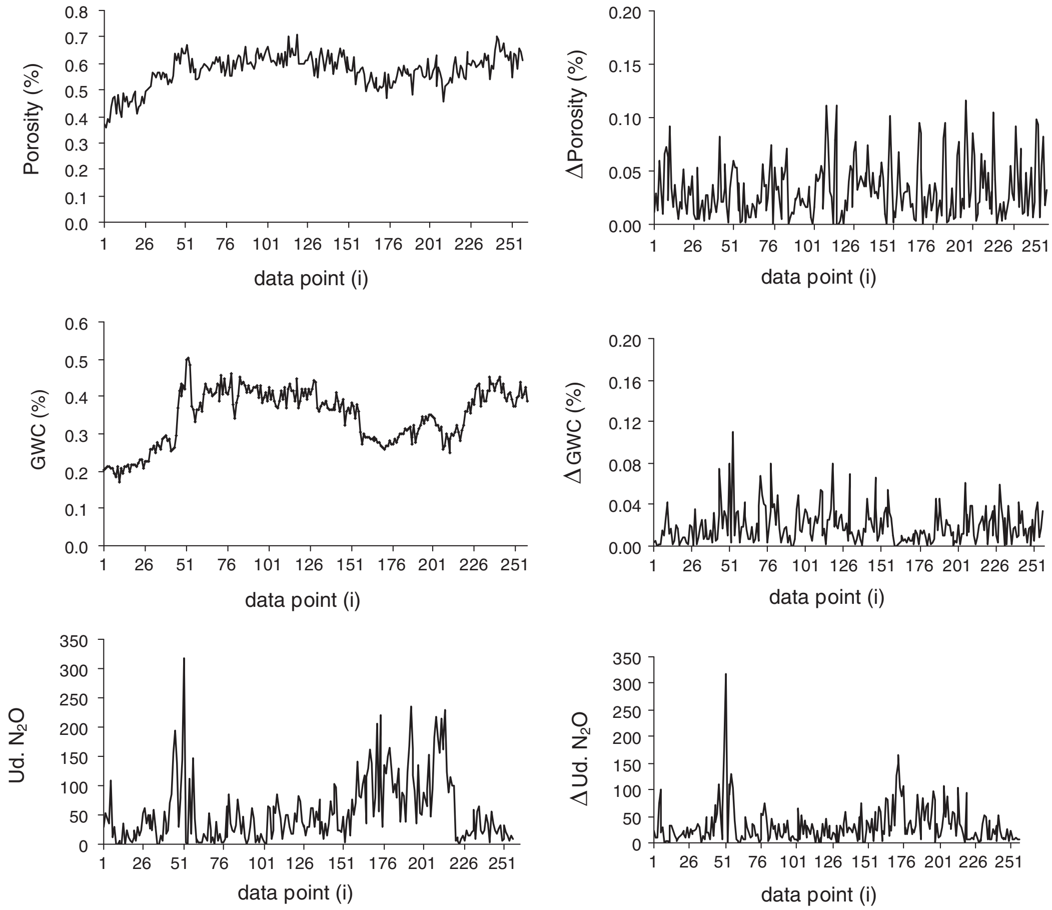

Fig. 1. Original data of the soil variables: Porosity (%), Gravimetric Water Content (GWC) (%) and N_2O flux on the left column. On the right side, the absolute differences obtained with lag 1 of the corresponding variable.

al., 1994; Schmitt et al., 1995; Taqqu et al., 1995). Applying these methods, researchers determined the fractal scaling properties and the long-range correlations in both stationary and non-stationary series.

In many soil studies, researchers have characterized the scaling property of a variable measured along a transect as a mass distribution on a spatial domain of the studied field (Zeleke and Si, 2004, 2006). For this characterization, the transect is divided into a number of self-similar segments. The differences among the subsets are identified using D(q) and a multifractal spectrum (Folorunso et al., 1994; Caniego et al., 2005; Tarquis et al., 2008a). Recently, several authors (Siqueira et al., 2013; Lopez de Herrera et al., 2016) have applied Multifractal Analysis to profiles of soil penetrometer resistance data sets and found that these methods added complementary information to describe the spatial arrangement to methods of classical statistics.

However, only recent works on agricultural soils have studied the application of these methods to cases in which a measure along a transect is observed as a random signal. Pozdnyakova et al. (2005) evaluated the spatial variability of cranberry yield by applying a Generalized Structure Function, proving the influence of multiscale factors (nonlinear structure functions). Kravchenko (2008) approached the spatial features of environmental and agronomic variables using multifractal characteristics in a stochastic simulation. Garcia Moreno et al. (2010) assessed the variability of soil surface roughness using the Generalized Structure Function of transects to compare soil types and tillage tools, with promising results.

In this work, we focused on the use of MFA to study the relation of the characterization of the measure among different scales. Comparing different methods, we found that there are several works studying wider scaling behaviours which cannot be captured in a consistent way by the MFA. These analyses include extended power-law scaling (linear relations between log structure functions of successive orders) at all lags, and frequency distributions of the variables' increments, which tend to be symmetric with peaks that grow sharper and tails that become heavier as the lags between pairs of values decrease (Guadagnini et al., 2015, 2014; Riva et al., 2015).

Based on the foregoing, the present study aimed to apply both types of MFA methods, the Generalized Structure Function and Multifractal Spectrum, to data on soil properties along a transect of arable fields, to compare and evaluate the results obtained for characterizing their structure and variability.

2. Material and methods

2.1. Experimental site

The data used in this paper were collected in a survey on a common 1024 m transect across arable fields at Silsoe in Bedfordshire,

Table 1
Descriptive statistics using the first fourth moments (average, variance, asymmetry and kurtosis) of soil porosity (Porosity), gravimetric water content (GWC) and N_2O flux values (N_2O).

Statistics	Porosity	GWC	N_2O
Average	0.5736	0.3475	54.42
Variance	0.0040	0.0055	2970.74
Asymmetry	−0.8559	−0.4289	1.59
Kurtosis	0.9440	−0.8398	2.81