Journal of Hydrology 553 (2017) 26–34

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Bed load transport for a mixture of particle sizes: Downstream sorting rather than anomalous diffusion

Niannian Fan, Yushu Xie, Ruihua Nie *

State Key Laboratory of Hydraulics and Mountain River Engineering, College of Water Resource & Hydropower, Sichuan University, Chengdu, Sichuan, China

ARTICLE INFO

Article history:
Received 17 April 2017
Received in revised form 2 July 2017
Accepted 8 July 2017
Available online 15 July 2017
This manuscript was handled by Corrado Corradini, Editor-in-Chief, with the assistance of Christophe Darnault, Associate Editor

Keywords:
Bed load tracers
Stochastic
Diffusion
Mixture
Ballistic regime
Downstream sorting

ABSTRACT

The stochastic nature of bed load transport induces diffusion of sediment tracers, which is governed by the dynamics of their bulk behavior over time. By deploying both numerical simulations and flume experiments, the emergent particle diffusion regimes for both uniform and mixed tracer particles were studied and compared. For uniform particles, power-law-distributed resting times T_r produced super-, sub- or normal diffusion regimes for certain values of the tail exponent v. Based on the assumption that heterogeneity in particle size leads to a power-law distribution of T_r, a completely different diffusion regime emerges in mixtures compared with those obtained from uniform particles with the same value of the tail exponent v. Mixtures exhibited the same ballistic regime (the variance of travel distance grows as time squared) for different values of v, and ballistic regimes for mixtures also emerged from several other tested models. Furthermore, our experimental results confirmed the ballistic regime; however, the decreasing number of tracked particles may result in apparent but deceptive sub-diffusion. We conclude that ballistic regimes for mixtures result from violations of the independent and identically distributed (i.i.d.) assumptions, attributing to downstream sorting processes.

1. Introduction

Detailed description of the dispersion of sediment particles in streams is essential for a better understanding of the physical processes which determine the bed stability, sediment texture, channel morphology, and stream ecology. Sediment transport is a fundamental factor for the design of hydraulic structure, flood management, navigation, and river restoration. However, predictions of sediment transport from existing sediment transport models have often yielded poor estimates (Gomez and Church, 1989; Hassan et al., 2008; Recking, 2013; Heyman et al., 2014) that have impacted sediment management decisions and affected stream function. Tracer particles may provide detailed information vital for understanding sediment transport processes and the development of predictive tools. However, the lack of detailed descriptions of particles' movement is attributed to technological limitations such as the inability to track large numbers of grains, and the focus on the mobility of bulk material (Nakagawa et al., 1982; Wilcock and McArdell, 1993; Parker, 2008; Houssais and Lajeunesse, 2012). In spite of the technical limitations on tracking particles, extensive researches have been conducted by using tracer particles in the field (Sayre and Hubbell, 1965; Drake et al., 1988; Ferguson and Wathen, 1998; Hassan et al., 1991, 2013; Schmidt and Ergenzinger, 1992; Habersack, 2001; Haschenburger, 2013; Phillips et al., 2013; MacVicar et al., 2015; Olinde and Johnson, 2015) and laboratory (Yang and Sayre, 1971; Todorovic, 1982; Wong et al., 2007; Hill et al., 2010; Roseberry et al., 2012; Martin et al., 2012; Fathel et al., 2015; Sun et al., 2015; Campagnol et al., 2015; Furbish et al., 2017), also, numerical simulations (Malmaeus and Hassan, 2002; Ganti et al., 2010; Bialik et al., 2012; Pelosi et al., 2014, 2016; Zhang et al., 2014; Fan et al., 2016a) were deployed.

The motions of individual particle on a stream bed consist of a series of step movements and rest periods (Einstein, 1937) under the conditions of either partial or full mobility (Wilcock and McArdell, 1993; Sun et al., 2015). The intermittent movements of individual particle are attributed to the irregular bed surface boundary, the wide range of particle sizes and shapes, bed surface arrangements and the turbulent nature of flow (Einstein, 1937; Hassan et al., 1991, 2013; Lajeunesse et al., 2010). Einstein (1937) was the first researcher to fully acknowledge the stochastic nature of sediment movement in streams and to incorporate the knowledge of bed surface boundary conditions and turbulence into sediment transport theory. Bed load movements in stream are stochastic even under quasi-steady conditions, narrowly graded

* Corresponding author.
E-mail addresses: fannian7172@126.com (N. Fan), scunie@163.com (R. Nie).

http://dx.doi.org/10.1016/j.jhydrol.2017.07.012
0022-1694/

Nomenclature

a	minimum possible value for a Pareto distribution
d	diameter of a particle [L]
k	number of steps of discrete time $t/\Delta t$ for computation of the inertia-mixture model
L_s	step length [L]
p	parameter for gamma distribution of particle size
r	lag Δt autocorrelation
t	time [T]
T_r	resting time [T]
T_s	step time [T]
x	travel distance in the streamwise direction[T]
β_x	exponent for diffusion; namely, the power-law exponent for variance of travel distance x with time
σ_x^2	the variance of the particle travel distance $[L^2]$
σ_s^2	the total step standard deviation of longitudinal travel distance $[L^2]$
σ_{sg}^2	the longitudinal travel distance variance associated with particle heterogeneity at one time step Δt $[L^2]$
$\Gamma(\cdot)$	gamma function
τ^*	Shields number
ν	tail exponent for power-law resting times T_r
ν'	tail exponent for power-law resting times for a mixture of particles

sediments and plane bed morphology (Bottacin-Busolin et al., 2008; Hill et al., 2010; Tregnaghi et al., 2012; Furbish et al., 2012a; Fan et al., 2014; Ancey et al., 2015). This inherent stochasticity leads to diffusion (also known as dispersion) of tracer particles over time (Martin et al., 2012; Fan et al., 2016a), which quantifies the dispersion of particles as they travel downstream, namely, the variance of particle travel distance σ_x^2 evolved over time t as

$$\sigma_x^2 = \langle (x_i - \langle x \rangle)^2 \rangle \propto t^{\beta_x}. \quad (1)$$

when $\beta_x = 1$, particle diffusion is "normal", i.e., the variance increases linearly with t. When $\beta_x \neq 1$, particle diffusion is collectively termed as "anomalous", and specifically, sub-diffusion for $\beta_x < 1$, super-diffusion for $1 < \beta_x < 2$, and ballistic-diffusion for $\beta_x = 2$ (Nikora et al., 2002; Martin et al., 2012; Furbish et al., 2012a). For ballistic-diffusion ($\beta_x = 2$), the leading and trailing edges of the tracers plume travel with the maximum and minimum particle velocities, respectively.

Stochastic theory provides the governing equations for bulk behaviors of stones conceptualized as statistically random phenomena (Einstein, 1937) when the determined statistical parameters of step lengths and resting times are known. The tail characters of resting times or step lengths particularly lead to different governing equations with different solutions and scaling rates. Early works on this topic presumed random step lengths and rest periods with assumed distributions (e.g., Einstein, 1937, and others). For normal dispersive ($\beta_x = 1$) models, the plumes diffuse linearly over time. Assuming a normal diffusive process, several thin tail distributions have been suggested to describe the travel distances of particles in both sand and gravel bed streams. The exponential distribution (Einstein, 1937; Sayre and Hubbell, 1965; Hassan et al., 1991; Ferguson and Wathen, 1998) and the gamma distribution (Yang and Sayre, 1971) are the most popular, as well as the Weibull distribution in more recent researches (Fathel et al., 2015).

Inspired by Nikora et al. (2002), recent studies have suggested that thin-tailed particle displacements may be an exception. In contrast to normal diffusion, anomalous diffusion takes place when a plume of sediment spreads either faster or slower than the normally dispersive plume, i.e., non-linearly spread. A thorough discussion of the normal/anomalous diffusion from thin/heavy tailed distributions of resting time for uniform bed load particles was provided by Fan et al. (2016a). For mixture particles, both Ganti et al. (2010) and Hill et al. (2010) demonstrated that a heavy-tail distribution can arise from an exponential step length and a gamma distribution of grain sizes. However, neither Ganti et al. (2010) nor Hill et al. (2010) performed numerical simulations or experiments to test if the emergent diffusion characters fit the predictions of their own theory.

Bradley et al. (2010) reanalyzed the travel distance data collected by Sayre and Hubbell (1965) from the North Loup River, a sand bed river. The reanalysis yielded a heavy-tailed distribution and the resulted super-diffusion model better fit the data than the classical step length models (i.e., the Sayre and Hubbell model). However, Bradley et al. (2010) did not consider the possibility that a heavy-tailed distribution could arise from the resting times rather than the step lengths. To explore the applicability of the heavy-tailed distribution of travel distances, Hassan et al. (2013) reanalyzed 64 data sets collected from gravel-bed rivers and study sites that covered a wide range of environment (e.g., arid, humid) and channel morphologies; they reported thin-tailed distributions in most examined cases but also questioned the methodology and suggested that future research should rely on real-time monitoring of the step lengths and resting times.

In contrast to Nikora et al. (2002), who asserted that heavy-tailed resting times are likely to result in anomalous sub-diffusion, Martin et al. (2012) and Phillips et al. (2013) observed an anomalous super-diffusion from heavy-tailed resting times. Zhang et al. (2012) provided a unified theory for bed load transport based on a tempered space-time random walk model. Voepel et al. (2013) empirically showed that the resting time of sediment in the channel was a first-passage time stochastic process, thus verifying the unified bed load theory of Zhang et al. (2012). Martin et al. (2014) used a mean-reverting random walk model to obtain the heavy-tailed resting times. However, neither Voepel et al. (2013) nor Martin et al. (2014) measured the resting time directly but rather used the changes in the initial position of a particle relative to the bed elevation. Fan et al. (2016a) studied the diffusion regimes for uniform particles and revealed that for the thin tailed velocities, the tail of the distribution of resting times instead of step lengths play the key role in determining diffusion regimes.

This study aims to explore diffusion characteristics both numerically and experimentally to resolve the motions of a large number of tracer particles for uniform and mixed bed material conditions. We numerically studied the diffusion of tracers based on our Episodic Langevin Equation (ELE) Model. Specifically, we focus on the framework of Ganti et al. (2010) and Hill et al. (2010) to determine if heavy-tailed distributions of resting time/step lengths for the mixtures could emerge from thin-tailed-distributed variables for uniform particles. Moreover, we conducted experiments to compare the results with the predictions of numerical models.