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Concentration distribution for pollutant dispersion in a reversal laminar flow

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ABSTRACT

Pollutant transport in reversal laminar flows gains its significance in various coastal regions. Since oscillation in the flow introduces much complexity into the transport process, little progress has been made to illustrate the evolution of concentration distribution. In this work, the first order expansion of the generalized dispersion model, as a simplified applicable method based on the previously proposed Aris-Gill expansion (Wang and Chen, 2016b,c), is applied to analytically study the pollutant dispersion in an open channel reversal laminar flow. This method is conveniently used to accurately predict the two-dimensional concentration evolution characteristic of peak concentration position and duration. The vertical concentration difference is determined to be tremendous and vary periodically, and the peak concentration appears at the freesurface or bottom depending on the reversal amplitude. The approach for vertical concentration to uniformity in the dispersion process lasts longer remarkably in reversal flows than that in steady flows.

1. Introduction

Associated with growing attentions for the tidal flats, estuaries and other coastal regions in regard of serious environmental and ecological problems, there is an increased demand on water quality models to accurately predict the transport of pollutant clouds once they are released into watercourses (Fischer, 1972, 1976; Fischer et al., 1979; Shucksmith et al., 2010; Zeng et al., 2012; Wu et al., 2012; Chen, 2013; Wang and Chen, 2015). A major concern goes to the accurate two-dimensional concentration distribution and its evolution in tidal or reversal flows, of essential significance to various applications such as environmental risk assessment and ecological restoration (Carvalho et al., 2009; US EPA, 1999; Wu and Chen, 2014b; Wu et al., 2015, 2016; Chen et al., 2015; Wang and Chen, 2015, 2016a, 2017; Fu et al., 2016).

The model of Taylor dispersion has been extensively applied in the study of pollutant transport. Taylor dispersion refers to the mechanism that solute spreads longitudinally under the combined action of transverse velocity nonuniformity and solute diffusion. The enhancement is so remarkable that the Taylor dispersivity, a virtual coefficient in a one-dimensional diffusion-like model for the dispersion process, is usually several orders of magnitude larger than the molecular diffusivity (Taylor, 1953). Following the canonical work of Taylor (1953), numerous works have been reported on the pollutant dispersion in reversal laminar flows.

Holley et al. (1970) firstly investigated the effects of transverse variations of velocity on dispersion in estuary flows, and found the dispersivity to be dependent of a characteristic parameter T_r representing the ratio of tidal period and the characteristic time needed for smearing out the vertical concentration nonuniformity by the diffusion effect. Fischer (1972) determined the amount of mass transported by a number of mechanisms involved in estuary flows, and noticed that the dispersivity can sometimes be negative for reversal flows. Then Smith (1982) modeled the periodic contracting and expanding of the concentration cloud in reversal flows, to explore the effect of the negative dispersivity. As described, the concentration cloud is contracted after each flow reversal when the characteristic time for smearing out the vertical concentration nonuniformity is comparable with or larger than the flow period. Yasuda (1984) introduced a different vertical-average for the definition of the dispersivity by considering the strong vertical shear of the flow movement in the initial stage, which can guarantee the positiveness of the dispersivity. Vedel and Bruus (2012), among others, as well focused on the dispersion in reversal flows in its transitional or steady stage (Aris, 1960; Chatwin, 1975; Fischer et al., 1979; Jimenez and Sullivan, 1984; Mazumder and Das, 1992; Hazra et al., 1996; Ng, 2004; Lee et al., 2014; Wang et al.,

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2015). While extensive researches on the pollutant transport in reversal flows mainly focused on the evolution of longitudinally distributed mean concentration, relatively little endeavor has been committed to the vertical concentration evolution.

Until very recently, Wu and Chen (2014a) for the first time analytically studied the evolution of transverse concentration distribution, in a case study for a tube flow, by a two-scale perturbation analysis. Then Wang and Chen (2016b) presented the full-time functions of transverse concentration distribution with the transitional effects of skewness and kurtosis fully analyzed using the developed Aris-Gill expansion. The transverse concentration is revealed with tremendous nonuniformity even in the asymptotic Taylor dispersion stage, and the time scale for transverse concentration to approach relative uniformity is determined to one-order of magnitude larger than that for the mean to approach longitudinal normality. This progress is further extended to describe the pollutant dispersion with wall absorption to characterize the extremely great transverse concentration difference (Wang and Chen, 2016c). However, all these progresses are confined to the fully-developed steady flows, with monotonous evolution under stationary velocity profile to reduce the transverse concentration nonuniformity. Time-varying and even reversal velocity profile of reversal laminar flows introduces much complexity to the transport process, the transverse concentration distribution and its approach to uniformity needs to be further explored.

This paper is to present a theoretical study of the two-dimensional concentration evolution in an open channel reversal laminar flow by the generalized dispersion model (Gill, 1967; Gill and Sankarasubramanian, 1970). As an alternative and simplified method based on the previously proposed Aris-Gill expansion in Wang and Chen (2016b,c), it brings convenient applications to predict the real two-dimensional concentration distribution of the pollutant cloud in reversal laminar flows in terms of peak concentration position and duration. The specific contents of this paper are: (I) to deduce the transport model for dispersion and give the reversal velocity profile; (II) to analytically derive the vertical concentration distribution functions and present the mean concentration distribution; (III) to analyze the vertical concentration distribution and its approach to uniformity, and to illustrate the effect of reversal amplitude.

2. Method

After the initial release of pollutant substance into the reversal laminar flow, the pollutant cloud driven by the oscillatory velocity moves at a time-varying speed and even reverses to the upstream direction during part of each period. The two-dimensional concentration evolution characterized by the peak concentration position and duration is still unknown for the case of a reversal flow. By the first order expansion of the generalized dispersion model, the transport characteristics can be essentially revealed with acceptable accuracy for the Taylor dispersion stage.

2.1. Formulation for scalar transport

The pollutant transport process in a fully-developed reversal laminar flow in open channel with water depth h, as shown in Fig. 1, is governed by the advection–diffusion equation as

$$\frac{\partial C^*}{\partial t} + u(t,z)\frac{\partial C^*}{\partial x} = D\frac{\partial^2 C^*}{\partial x^2} + D\frac{\partial^2 C^*}{\partial z^2}, \tag{1}$$

where C^* is the concentration, t the time, u the reversal flow velocity, z the lateral coordinate, x the longitudinal coordinate, and D the molecular diffusivity. It should be noted that the present sketch is the most typical one that could represent the configuration of bank-effect channel flow, in which the flow regime is symmetrical about the centreline.

Consider the initial condition as a uniform and instantaneous release of scalar substance with mass Q at $x = 0$ as

$$C^*(t,x,z)|_{t=0} = \frac{Q}{h}\delta(x), \tag{2}$$

where $\delta(x)$ is the Dirac delta function.

Since the amount of released substance is finite, the upstream and downstream boundary conditions are

$$C^*(t,x,z)|_{x=\pm\infty} = 0. \tag{3}$$

The non-penetration condition at the bottom and free surface read as

$$\left.\frac{\partial C^*(t,x,z)}{\partial z}\right|_{z=0} = \left.\frac{\partial C^*(t,x,z)}{\partial z}\right|_{z=h} = 0. \tag{4}$$

For simplicity, dimensionless variables are introduced as

$$\tau = \frac{Dt}{h^2},\ \zeta = \frac{z}{h},\quad \eta = \frac{x - \langle\overline{u}\rangle t}{h},\quad \psi = \frac{u - \langle\overline{u}\rangle}{\langle\overline{u}\rangle},$$
$$C = \frac{h^2}{Q}C^*,\quad Pe = \frac{\langle\overline{u}\rangle h}{D}, \tag{5}$$

where τ represents the time, ζ the vertical coordinate, η the longitudinal coordinate, ψ the velocity deviation, C the concentration, Pe the Péclet number, and $\langle\overline{u}\rangle$ is the periodically and vertically averaged velocity, with the overline indicating the operation of vertical average and the bracket indicating the operation of reversal period average for a variable ϱ defined respectively as

$$\langle\overline{\varrho}\rangle \equiv \frac{1}{T}\int_0^T\int_0^1 \varrho\, d\zeta dt, \tag{6}$$

where T is the reversal period.

Then the governing equation and the initial and boundary conditions are rewritten as

$$\frac{\partial C}{\partial \tau} + Pe\psi(\tau,\zeta)\frac{\partial C}{\partial \eta} = \frac{\partial^2 C}{\partial \eta^2} + \frac{\partial^2 C}{\partial \zeta^2}, \tag{7}$$

$$C(\tau,\eta,\zeta)|_{\tau=0} = \delta(\eta), \tag{8}$$

$$\left.\frac{\partial C(\tau,\eta,\zeta)}{\partial \zeta}\right|_{\zeta=0} = \left.\frac{\partial C(\tau,\eta,\zeta)}{\partial \zeta}\right|_{\zeta=1} = 0, \tag{9}$$

$$C(\tau,\eta,\zeta)|_{\eta=\pm\infty} = 0. \tag{10}$$

2.2. Tidal velocity profile

This work focuses on the essential mechanism of dispersion in reversal laminar flows. To enable the completely analytical exploration, a simplified flow case with fixed freesurface elevation is introduced (Fischer et al., 1979; Yasuda, 1984; Hazra et al., 1996; Jansons et al., 2006; Ng, 2006; Wu et al., 2012; Zeng et al., 2012; Wang and Chen, 2015). For open channel flows, the basic equation for momentum is adopted as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}, \tag{11}$$

where the pressure gradient $-\partial P/\partial x$ with a time period of T can be expressed by Fourier series (Wu et al., 2012) as

$$-\frac{\partial P}{\partial x} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi t}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi t}{T}\right). \tag{12}$$

and