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Stream depletion rate for a radial collector well in an unconfined aquifer near a fully penetrating river

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ABSTRACT

The stream depletion rate (SDR) is computed for a radial collector well installed in a semi-infinite, anisotropic, homogeneous, unconfined aquifer near a fully penetrating stream with a streambed with reduced conductivity. For small pumping rates dimensional analysis and other arguments allow the SDR to be expressed as a function of eight parameters that describe the effect of properties of the aquifer and streambed as well as the configuration and placement of the well. The calculations employ some results from Huang et al. (2012, *J. Hydrol.*), who expressed the SDR as a quintuple integral, but by computing four of the integrals analytically, the present solution requires less computational effort. Analytical calculation shows that the SDR in steady state does not depend on the streambed properties (or any other parameters): Given enough time, the flow through the streambed will equal the pumping rate of the well. Values of SDR are supported by comparing to previous solutions for special cases corresponding to appropriate limiting values of the parameters. Effects of the eight dimensionless parameters are studied systematically: The properties of the streambed, anisotropy of the aquifer, and the position of the well affect the SDR more strongly than the orientation, length, and depth of the laterals.

1. Introduction

Wells located in high yielding aquifers next to water bodies exploit the natural filtration by the soil and blending with groundwater to produce large volumes of water of acceptable quality. The quantity and quality of water produced by riverbank filtration systems varies because of the dynamic nature of the connection between the surface water and groundwater. To estimate the quality of the effluent and evaluate the effect of pumping on local and regional water budgets, producers and regulators use the stream depletion rate (SDR)

$$SDR = \frac{q_s}{Q} \tag{1}$$

or the ratio of the flow q_s from the stream and the pumping rate Q of the well. We evaluate the effect of several parameters on the SDR for a radial collector well in an unconfined aquifer so that radial collector wells can be designed and operated more effectively.

Dimensional analysis helps to identify the parameters that control the SDR for a radial collector well in a homogeneous, anisotropic, unconfined aquifer (Fig. 1). If the aquifer is infinitely long in the y-direction, the flow from the stream depends on the rate Q and duration t of pumping; the streambed hydraulic conductivity K' and thickness b'; aquifer properties including the saturated thickness H, horizontal hydraulic conductivity K_x (or transmissivity $T = K_xH$), vertical hydraulic conductivity K_z, specific yield S_y, and specific storage S_s (or storage coefficient $S = S_sH$); distance L_x from the stream to the caisson; and the length ℓ_i, vertical position L_z, and orientation θ_i of the laterals. The relationship can be simplified by realizing that the streambed parameters can be combined in a conductance coefficient (Hantush, 1965) and the specific yield, specific storage, and saturated thickness can be combined as the ratio S_y/S. Then, dimensional analysis yields

$$\begin{aligned} \mathrm{SDR} &= f\left(\frac{Tt}{SL_x^2}, \frac{K'L_x}{K_xb'}, \theta_i, \frac{\ell_i}{H}, \frac{K_z}{K_x}, \frac{Q}{K_xH^2}, \frac{S_y}{S}, \frac{L_x}{H}, \frac{L_z}{H}\right) \\ &= f(\tau, \chi, \kappa_z, \Lambda_i, \theta_i, \varepsilon, \gamma, \rho_x, \rho_z) \end{aligned} \tag{2}$$

The second equality in Eq. (2) defines several dimensionless parameters to be considered (Table 1).

Previous work on this problem, which is summarized in Table 2, offers insight into the effects of the parameters in the dimensional analysis on SDR. For a fully penetrating vertical well in an aquifer near a fully penetrating stream, Theis (1941) and Glover and Balmer (1954) neglected vertical flow and found the SDR to be

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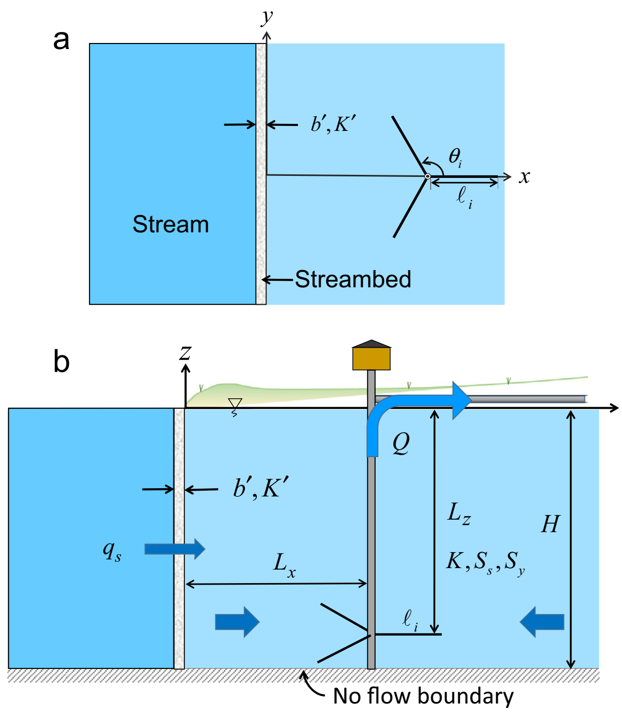

Fig. 1. Conceptual model for a radial collector well installed in an anisotropic, unconfined aquifer adjacent to a fully penetrating stream: (a) plan view, (b) profile view.

Table 1
Dimensionless parameters affecting the stream depletion rate.

Parameter	Description
$\tau = \frac{Tt}{SL_x^2}$	Time normalized by hydraulic diffusion time
$\chi = \frac{K'L_x}{K_x b'}$	Streambed conductance coefficient
$\kappa_z = \frac{K_z}{K_x}$	Anisotropy parameter: ratio of vertical and horizontal conductivities
$\Lambda_i = \frac{\ell_i}{H}$	Dimensionless length of ith lateral
θ_i	Angle between the ith lateral and positive x-axis
$\varepsilon = \frac{Q}{K_x H^2}$	Well strength
$\gamma = \frac{S_y}{S}$	Ratio of specific yield and storage coefficient
$\rho_x = \frac{L_x}{H}$	Dimensionless distance between stream and caisson
$\rho_z = \frac{L_z}{H}$	Dimensionless depth of laterals (defined as positive)

$$\mathrm{SDR}'_T = \mathrm{erfc}\left(\frac{1}{2\tau^{1/2}}\right) \tag{3}$$

Initially the SDR is small because the flow to the well is from the aquifer. As time passes, or τ increases, the cone of depression expands to include the streambed, and SDR increases. At large times, SDR approaches one, and the flow to the well is supplied entirely by the stream. The change in SDR depends on the diffusion time scale $L_x^2/(T/S)$ (Jenkins, 1968); changes in SDR take longer for greater distances between the well and the stream, less conductive aquifers, or aquifers in which more water is released per unit change in head.

Several models have allowed for a streambed with reduced permeability. For a case with negligible vertical flow, the SDR depends on the conductance coefficient $\chi = K'L_x/K_x b'$, which characterizes the flow processes associated with the streambed (Hantush, 1965):

$$\mathrm{SDR}'_H = \mathrm{erfc}\left(\frac{1}{2\tau^{1/2}}\right) - \exp\left(-\frac{1}{4\tau}\right)\mathrm{erfcx}\left(\frac{1}{2\tau^{1/2}} + \chi\tau^{1/2}\right) \tag{4}$$

where erfcx( ) is the scaled complementary error function, defined as $\mathrm{erfcx}(z) = \exp(z^2)\mathrm{erfc}(z)$. Small χ, which corresponds for example to a thick streambed with low conductivity, leads to smaller SDR. However, for any $\chi > 0$, the SDR approaches one given enough time; that is, even for streambeds with small conductivity, all of the flow to well will eventually come from the stream. This observation differs from results for a radial collector well in an unconfined, anisotropic aquifer with vertical flow, which show that the steady-state SDR decreases as K'/K_x decreases (Huang et al., 2012, hereafter HTY). However, steady SDR of less than one must be incorrect because a water balance applied to a semi-infinite aquifer shows that in steady state all of the well's flow must come from the stream.

The temporal evolution of the SDR becomes more complicated when gravity drainage supplies the well. Analytical solutions for both a semi-infinite aquifer (HTY) and an aquifer bounded by two streams and two no-flow boundaries (Huang et al., 2016a) predict an intermediate stage in which delayed yield from the aquifer (Neuman, 1972) maintains constant SDR. The unsteady behavior of SDR can be important in practice when times to steady state are large. For example, the Theis solution in Eq. (3) shows that SDR = 0.99 when $\tau = 3910$, and the Hantush solution in Eq. (4) shows that when streambed conductance is included, the time to steady state increases. The result from Eq. (3) applied to wells near the Cedar River in Iowa (Turco and Buchmiller, 2004) indicates that the time to steady state ranges from 1 d for wells close to the river in highly conductive soil to 3100 d for wells farther away in less conductive soil. Pumping tests to determine aquifer properties typically last less than 3 d (Chin, 2006, p. 744), and the post-construction tests of two radial collector wells near the Des Moines River lasted 3 and 4 d (Moore et al., 2012), while a test of a radial collector well in the Tailan River Basin included 16 d of pumping (Appiah-Adjei et al., 2012). Therefore, understanding the unsteady behavior of SDR can help in interpreting data collected during short-term pumping tests, especially if water quality data are included.

Previous work allows the effect of several of the other parameters in Eq. (2) on SDR to be determined in cases with vertical flow. The value of the SDR in the intermediate stage depends on anisotropy, or the ratio of vertical and horizontal hydraulic conductivities, i.e., $\kappa_z = K_z/K_x$: As κ_z decreases, gravity drainage decreases, and more flow comes from the stream (Huang et al., 2016a). Lateral configuration affects SDR less than anisotropy. Lateral configurations that reduce the distance between the well and the stream—either by increasing the length of laterals pointed toward the stream or by orienting more laterals toward the stream—produce a slightly higher surface water percentage (i.e., SDR) before steady state is reached (Moore et al., 2012).

The last four parameters in Eq. (2) have not been studied in detail for flow to radial collector wells. As shown in Section 2.1, the parameter $\varepsilon = Q/K_x H^2$ determines whether the drawdown is large enough for nonlinear effects to be important. The parameter $\gamma = S_y/S$ determines the duration of the intermediate stage with constant drawdown (and presumably SDR); as γ increases, the duration of the intermediate stage also increases for a well pumping in an infinite unconfined aquifer (Neuman, 1975). The two ratios of length scales characterize the location of the well. Locating the well closer to the stream, or reducing $\rho_x = L_x/H$, increases the fraction of the well's flow that comes from the stream (Moore et al., 2012). Under certain conditions locating the laterals of a radial collector well deeper in the aquifer can increase SDR (Huang et al., 2016a).

In this paper the HTY solution is re-evaluated to investigate the causes of differences between SDR in aquifers with and without vertical flow and to quantify the effects of the parameters from