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A method of estimating sequential average unsaturated zone travel times from precipitation and water table level time series data

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ABSTRACT

A method to estimate sequential average unsaturated zone travel time with high temporal resolution has been developed. The method is built upon the conventional cross-correlogram analysis, while the estimation errors are significantly reduced by the proposed schemes. In addition, an analytical relationship between the estimated travel time and the corresponding parameter of a physically-based water table (WT) fluctuation model has been newly established. For validation, applications were performed using WT and precipitation data from two locations with contrasting properties. The method was found to derive distinct characteristics in the estimated travel time, which reflect the unsaturated hydraulics by estimating large means and variations in travel times for low permeability unsaturated zones; whereas, the values are typically small for highly permeable unsaturated zones. The overall results suggest that the proposed method can be potentially adopted to complement other methods in the assessment of groundwater vulnerability to surface contaminants and the hydraulic characterizations of unsaturated zones.

1. Introduction

Residence or travel time of percolating water fluxes in or through unsaturated (vadose) zones provides useful information on unsaturated hydraulics and associated processes. A sound understanding of the unsaturated processes has practical importance in the quantitative and qualitative assessments of groundwater resources, such as groundwater recharge rate and potential vulnerability to surface contaminants (Sousa et al., 2013). However, the investigation of the travel time is not readily achievable due to the complexities in the permeability and storage capacity of an unsaturated zone (UZ) containing spatial heterogeneities (Dunn et al., 2007; Mattern and Vanclooster, 2010; Sousa et al., 2013). Due to this difficulty, various studies have been proposed to investigate the travel time based on natural or artificial tracers (Cook et al., 1994; Edmunds and Smedley, 2000; Caschetto et al., 2016), field and laboratory experiments (Cooper et al., 1990; Izbicki et al., 2000; Ireson et al., 2009), and physically-based models (Mathias et al., 2006; Harman et al., 2011; Sprenger et al., 2016).

With the increased availability of groundwater data, time series analysis has been one of the preferred options for investigating the travel time (e.g., Lee et al., 2006; McCoy and Blanchard, 2008; Okkonen and Kløve, 2010; Delbart et al., 2014; Neto et al., 2015). The vast majority of the studies have estimated the travel time based on the cross-correlogram analysis of precipitation and the resulting WT responses. In the earlier studies, only a single representative travel time was obtained aggregated for a whole period of the time series data. However, in actual groundwater data, the timing and pattern of WT responses to the precipitation show significant variability according to weather conditions. Therefore, it is obvious that the transiently estimated travel time provides richer information on the hydraulics of the UZ. To estimate temporally varying travel time, a few studies adopted a moving temporal window with finite size in the cross-correlogram analysis (Lee et al., 2006; Bailly-Comte et al., 2011; Delbart et al., 2014). Among these studies, Delbart et al. (2014) have acquired a temporally variable

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travel time based on cross-correlograms over a window of approximately 90 days, from which the impact of different precipitation patterns and hydrogeological conditions on the variability of travel time in a few UZs was investigated. However, in the previous studies, the detailed evolution of the travel time according to seasonal changes were not fully accounted for due to the low temporal resolution. In addition, the linkage of the estimated travel time to a physically-based model responsible for delayed drainage within an UZ, which may provide physical insights regarding the hydraulics of the UZ, has rarely been made.

In the present study, a method to estimate the average unsaturated zone travel time (AUZT) of percolating water fluxes through vadose zones since the start of rainfall sequentially with high temporal resolution has been developed; this capability has not been available from previous methods. In addition, an analytical relationship between the estimated AUZT and a physically-based model accounting for delayed drainage within an UZ was investigated. To validate the developed method, applications were performed using actual WT level and precipitation time series data from two locations in South Korea with contrasting hydraulic characteristics, and the impacts of the UZ and the weather conditions on the estimated AUZT were analyzed.

2. Methodology

2.1. Determination of transient AUZT

In the present study, it is assumed that the average time (duration) of infiltrated water percolating through an UZ can be approximated between the start of a precipitation event and the corresponding peak WT responses. In the development of the method, it is also assumed that there are no missing observations in the WT data. In the present study, cross-correlogram analysis of precipitation and the WT time series is adopted to determine AUZT by using sequential movement of the data window with a finite size. The transient AUZT with high temporal resolution can be estimated using sequential movement of a small window, from which the transient unsaturated hydraulics can be understood. For this purpose, a 30-day temporal window moving daily in time was adopted. The size of the temporal movement of the window in previous studies (e.g., Lee et al., 2006; Delbart et al., 2014) was typically a few months, which masked detailed evolution of transient AUZTs due to the low resolution.

As a method for reducing errors in AUZT estimations with high resolution, discrete precipitation data is converted to continuous pseudo water level data (η) using a modified version of the zero-lag-time WT fluctuation model proposed by Park and Parker (2008, Eq. (16))

$$\eta_{i+1} = \eta_i \exp(-k\Delta t) + \frac{p_i}{k}[1 - \exp(-k\Delta t)] \tag{1}$$

where p_i is the precipitation rate [LT^{-1}] at the ith time-step (typically day), η_{i+1} and η_i are the pseudo water levels [L] at the $i+1$th and ith time-steps, respectively, and Δt is the time segment of temporal discretization. In Eq. (1), it is assumed that precipitation is the only cause of WT fluctuation, where other causes, such as evapotranspiration, are disregarded, dictating that the methodological application is restricted accordingly. The parameter k is the exponential time constant [T^{-1}] regarding the rate of WT recession in time, whose value is proportional to regional hydraulic conductivity (Park et al., 2011). From our experiments, it was found that time lag determination by the proposed method works best when k is equivalent to the value representing the recession rate of WT data of interest ($\mathbf{h}$). As an additional error reduction strategy, an AUZT is determined only when the last time step of the window has effective precipitation exceeding a predefined rate (half of the mean daily precipitation in the present study). In this manner, an AUZT determination from inconsequential correlation peaks due to WT fluctuation extraneous to precipitation is excluded.

Based on the above schemes, the cross-correlogram ($\mathbf{r}_l$) for a window with length W at the lth time step is obtained from the following equations:

$$r_l(m) = \frac{(\boldsymbol{\eta}_l - \overline{\boldsymbol{\eta}}_l)^{\mathrm{T}}(\mathbf{h}_{l+m} - \overline{\mathbf{h}}_{\mathbf{l+m}})}{\sigma_{\boldsymbol{\eta}}\sigma_{\mathbf{h}}} \tag{2}$$

$$\boldsymbol{\eta}_l = [\eta_{1+(l-1)}, \ldots, \eta_{W+(l-1)}]^{\mathrm{T}} \tag{3}$$

$$\mathbf{h}_{l+m} = [h_{1+(l-1)+m}, \ldots, h_{W+(l-1)+m}]^{\mathrm{T}} \tag{4}$$

where lag m varies from zero to half of the window size (i.e., $m = 0, \ldots, W/2$), $\overline{\boldsymbol{\eta}}_l$ and $\mathbf{h}_{l+m}$ are the mean vectors of $\boldsymbol{\eta}_l$ and $\mathbf{h}_{l+m}$, and $\sigma_{\boldsymbol{\eta}}$ and $\sigma_{\mathbf{h}}$ are the standard deviations of $\boldsymbol{\eta}_l$ and $\mathbf{h}_{l+m}$, respectively. From Eqs. (2)–(4), the time step corresponding to the largest correlation coefficient is determined as the AUZT for the lth window (t'_l) as

$$t'_l = \mathrm{argmax}_t \mathbf{r}_l(t) \tag{5}$$

In the above equation, only the t'_l, whose coefficient $r_l(t'_l)$ is larger than ϵ, is considered as a meaningful result, where ϵ is the root of

$$f = \sqrt{2}\mathrm{erf}^{-1}(CL) - r\sqrt{\frac{(N-2)}{1-r^2}} \tag{6}$$

and CL is the confidence level (Diggle, 1990).

In the practical application of the proposed method, a finite window size (W = 30 days) is used, and the time lag showing the maximum correlation value is determined as the AUZT. In the highest peak selection, the peak value, with a correlation coefficient over 0.2968, and corresponding to a 90% significance level (Eq. (6), is considered a reliable estimation (Lee et al., 2006; Delbart et al., 2014). If there are no peaks with correlation coefficients over 0.2968, the AUZT of the selected window is regarded as indeterminable.

2.2. Delayed WT fluctuation model

Although the proposed method can provide sequential AUZT with high temporal resolution, the method is not applicable to sparsely measured WT data. Recently, Jeong and Park (2017) proposed a WT fluctuation model (hereafter called the JP model) that can consider delayed recharge flux in the UZ. Potentially, the model can be calibrated to sparsely measured WT level data. The change in WT can be approximated using both the recharge by precipitation and net groundwater flux as (Park and Parker, 2008),

$$\frac{dh}{dt} = -kh + \frac{R}{\phi} \tag{7}$$

where h is the discharge head [L] representing the groundwater storage within a unit aquifer area [L^2], t is the time [T], ϕ is the fillable porosity [–], and R is the recharge rate [LT^{-1}]. The recharge rate R, impeded within the UZ, can be approximated by employing the delayed yield model (Boulton, 1954)

$$R = \alpha \int_0^t \Phi(\tau) \exp[-\alpha(t-\tau)] d\tau \tag{8}$$

which is often used in well hydraulics to model delayed drainage due to a recessed WT (Moench, 1997). In the above equation, α is the shape parameter [T^{-1}] regarding water flux dispersion and slowness of percolation. The source function $\Phi(t)$ indicates the amount of water infiltrating the surface between 0 and Δt, with