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Research papers

Encounter risk analysis of rainfall and reference crop evapotranspiration in the irrigation district

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ABSTRACT

Rainfall and reference crop evapotranspiration are random but mutually affected variables in the irrigation district, and their encounter situation can determine water shortage risks under the contexts of natural water supply and demand. However, in reality, the rainfall and reference crop evapotranspiration may have different marginal distributions and their relations are nonlinear. In this study, based on the annual rainfall and reference crop evapotranspiration data series from 1970 to 2013 in the Luhun irrigation district of China, the joint probability distribution of rainfall and reference crop evapotranspiration are developed with the Frank copula function. Using the joint probability distribution, the synchronous-asynchronous encounter risk, conditional joint probability, and conditional return period of different combinations of rainfall and reference crop evapotranspiration are analyzed. The results show that the copula-based joint probability distributions of rainfall and reference crop evapotranspiration are reasonable. The asynchronous encounter probability of rainfall and reference crop evapotranspiration is greater than their synchronous encounter probability, and the water shortage risk associated with meteorological drought (i.e. rainfall variability) is more prone to appear. Compared with other states, there are higher conditional joint probability and lower conditional return period in either low rainfall or high reference crop evapotranspiration. For a specifically high reference crop evapotranspiration with a certain frequency, the encounter risk of low rainfall and high reference crop evapotranspiration is increased with the decrease in frequency. For a specifically low rainfall with a certain frequency, the encounter risk of low rainfall and high reference crop evapotranspiration is decreased with the decrease in frequency. When either the high reference crop evapotranspiration exceeds a certain frequency or low rainfall does not exceed a certain frequency, the higher conditional joint probability and lower conditional return period of various combinations likely cause a water shortage, but the water shortage is not severe.

1. Introduction

Water shortage in the irrigation districts has a considerable effect on the food security and agricultural development. Agricultural freshwater withdrawal is about 70% of the total amount. Irrigated agriculture accounts for 40% of global food production (Shiklomanov, 1997; Döll and Siebert, 2002; Muralidharan and Knapp, 2009). In the semi-arid region of northwest China, water shortage in the irrigation district poses a threat to agriculture production (Zhou and Li, 2003). Scholars have examined water issues such as optimal water allocation (Rosenberg et al., 2003; Lu et al., 2011; Li et al., 2015), water balance analysis (Ji et al., 2007; Roost et al., 2008), and irrigation water demand and supply (Araya and Stroosnijder, 2011; Zhang et al., 2013), but until now, the water shortage characteristics in the irrigation district cannot be fully revealed.

Water shortage is caused by the imbalance of water supply and water demand. For the irrigation district, the water supply mainly refers to rainfall and irrigation water, and the water demand involves crop water requirement. Commonly, rainfall indicates the natural water supply and irrigation water denotes the artificial water supply. The reference crop evapotranspiration (ET_0) is a key parameter of crop water requirement and can be used to represent the water demand (Allen et al., 1998). Under natural water supply conditions, rainfall and ET_0 are two related random hydrologic variables of the weather system with relevance for irrigation management and planning. Many related studies have indicated that

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the spatio-temporal characteristics of rainfall and ET_0 can explain the changes of the natural water supply and demand in the irrigation district (Katerji and Rana, 2006; Paulo and Pereira, 2007a, 2007b; Tabrizi et al., 2010; Vangelis et al., 2011). The correlations between rainfall and ET_0 are also explored to forecast and estimate their change trends or characteristics (Zhang et al., 2015).

Rainfall and ET_0 are the natural water supply and water demand in the irrigation district, respectively. Therefore, the joint distribution characteristics of rainfall and ET_0 can clarify the static characteristics of the natural water supply and demand, which can provide the reference for water shortage risk evaluation in the irrigation district. However, studies on the change laws and statistical characteristics of rainfall and ET_0 consider their correlations to be insufficient, whereas the encounter risk of rainfall and ET_0 remains unexplored. Only Ding et al. (2011) and Zhang et al. (2014) construct the joint probability distribution (JPD) of rainfall and ET_0 with the copula method.

The construction of the joint probability distribution of rainfall and ET_0 can describe the encounter risk of the natural water supply and demand and estimate various encounter probabilities, conditional probabilities and return periods with different magnitude combinations of rainfall and ET_0. In practice, rainfall and ET_0 may have different marginal distributions and their relations are nonlinear. Although the conventional joint distribution (such as the exponential distribution, the gamma distribution, the gumbel mixed distribution, et al.) can reveal the correlations of two variables, both variables are required to obey the same probability distribution (Coles and Tawn, 1991; Yue et al., 1999; Goel et al., 2000; Yue et al., 2001; Yue, 2001a, 2001b; Yue and Wang, 2004; Han et al., 2007; Liu and Chen, 2009). To address this problem, the copula function is used with its flexibility of univariable marginal distributions and joint link function. Currently, the copula method is prevalent in the hydrological field of drought (Francesco et al., 2009; Serinaldi et al., 2009; Salvadori and De Michele, 2015; Zhang et al., 2015), sea storm analysis (De Michele et al., 2007; Krzysztof et al., 2014), flood risk analysis (Sunyer et al., 2009; Grimaldi and Serinaldi, 2006; Tatiana et al., 2012), tail dependence (Huang et al., 2016; Di Bernardino and Rullière, 2016), and streamflow (Chen et al., 2015; Changsam and Taesam, 2015). However, its application in water shortage that results from water supply and demand remains to be studied.

In this study, based on the annual rainfall and ET_0 data series from 1970 to 2013 in the Luhun irrigation district of China, the statistical characteristic and marginal distribution of the natural water supply and demand are presented. Then with the copula function, the joint probability distribution of rainfall and ET_0 is established. Different synchronous-asynchronous encounter situations of the annual rainfall and ET_0 are analyzed. Finally, the conditional probability and return period of rainfall and ET_0 with different magnitudes are explored.

2. Mathematical methods

2.1. Copula method

Sklar theorem (Sklar, 1959) is the theoretical basis of the copula method. A bivariate copula function is the joint uniform distribution defined as a mapping C: $[0,\ 1]^2 \to [0,\ 1]$ (Nelsen, 1999). Assume that there are two continuous random variables denoted as X and Y. Let $F_X(x)$ and $F_Y(y)$ be the marginal distributions of X and Y and $F(x, y)$ be the joint distribution. If $F_X(x)$ and $F_Y(y)$ are continuous, there is a uniquely determined copula function $C_\theta(u, v)$ as:

$$F(x,y) = C_\theta(F_X(x), F_Y(y)), \forall x, y \tag{1}$$

where $C_\theta(u, v)$ is called the copula function and θ is a parameter to be determined.

The symmetrical Archimedean copula family with parameter θ is the most popular in hydrology and water resources. Kendall's correlation coefficient (Davis and Chen, 2007) τ describes the nonlinear correlation of variables, which is estimated using the following formula:

$$\tau = 1/C_n^2 \sum_{i<j} sign\left[(x_i - x_j)(y_i - y_j)\right] \tag{2}$$

$$sign\left[(x_i - x_j)(y_i - y_j)\right] = \begin{cases} 1 \\ 0 \\ -1 \end{cases} \begin{cases} (x_i - x_j)(y_i - y_j) > 0 \\ (x_i - x_j)(y_i - y_j) = 0 \\ (x_i - x_j)(y_i - y_j) < 0 \end{cases} \tag{3}$$

Table 1 shows three popular Archimedean copula representations and the relation between Kendall's τ and parameter θ.

2.2. Identification and goodness-of-fit evaluation of the copula function

The appropriate copula is identified by Kolmogorov-Smirnov (K-S) test. The identified statistics is given as

$$D = \max_{1 \leqslant k \leqslant n} \{|C_k - m_k/n||C_k - (m_k - 1)/n|\} \tag{4}$$

where C_k is the value of the observed pairs (x_k, y_k) of the copula function; m_k is the number of observed pairs (x_k, y_k) that satisfy $x \leqslant x_k$ and $y \leqslant y_k$; D is the identified statistic.

The ordinary least square (OLS) simply is applied to evaluate the goodness-of-fit of the copula function as follows:

$$OLS = \sqrt{(1/n)\sum_{i=1}^{n}(P_i - P_{ei})^2} \tag{5}$$

where P_i and P_{e_i} are the calculated value and empirical value of the joint probability distribution respectively. According to Ding et al. (2011), P_e is calculated as:

$$P_e(x_i, y_j) = P(X \leqslant x_i, Y \leqslant y_j) = \sum_{m=1}^{i}\sum_{k=1}^{j} n_{m,k}/(N+1) \tag{6}$$

where N is the number of observed pairs (x_k, y_k); $n_{m,k}$ is the number of observed pairs (x_k, y_k) that are less than or equal to pairs (x_i, y_j).

Thus, the established procedure of bivariate joint distribution based on the copula function is the following: (1) $F_X(x)$ and $F_Y(y)$ are estimated using the appropriate cumulative distribution function; (2) Kendall's τ and parameter θ are obtained from Table 1 to establish the bivariate joint distribution function; (3) the most

Table 1
Three popular used Archimedean copula representations.

Archimedean copula	$C_\theta(u, v)$	Relation between τ and θ	θ's range
Frank	$-\frac{1}{\theta}\ln\left[1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right]$	$\tau = 1 - \frac{4}{\theta}\left[-\frac{1}{\theta}\int_\theta^0 \frac{t}{\exp(t)-1}dt - 1\right]$	$\theta \in R$
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$\tau = \frac{\theta}{\theta+2}$	$\theta > 0$
Gumbel-Hougaard	$\exp\left[-((-\ln u)^\theta + (-\ln v)^\theta)^{1/\theta}\right]$	$\tau = 1 - \frac{1}{\theta}$	$\theta \geqslant 1$