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Technical Note

Dynamic flow-driven erosion – An improved approximate solution

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ABSTRACT

Rose et al. (2007) published an approximate solution of dynamic sediment concentration for steady and uniform flows, and this approximate solution shows a peak sediment concentration at the early stage of a runoff event, which can be used to describe and explain the first flush effect, a commonly observed phenomenon, especially in the urban environment. However the approximate solution does not converge to the steady state solution that is known exactly. The purpose of the note is to improve the approximate solution of Rose et al. (2007) by maintaining its functional form while forcing its steady state behaviour for sediment concentration to converge to the known steady state solution. The quality of the new approximate solution was assessed by comparing the new approximate solution with an exact solution for the single size class case, and with the numerical solution for the multiple size classes. It was found that 1) the relative error, or discrepancy, decreases as the stream power increases for all three soils considered; 2) the largest discrepancy occurs for the peak sediment concentration, and the average discrepancy in the peak concentration is less than 10% for the three soils considered; 3) for the majority of the 27 slope-flow combinations and for the three soils considered, the new approximate solution modestly underestimates the peak sediment concentration.

1. Introduction

Soil erosion and sediment delivery are highly dynamic. Following the on-set of a runoff event, there would usually be a surge in sediment concentration and concentrations of other pollutants, even if the flow is essentially steady. The initial high level of concentration is often referred to as the first flush effect, which is particularly pronounced in urban environments (e.g. Deletic, 1998; Barco et al., 2008; Kayhanian et al., 2012), although this first flush phenomenon and the resulting clockwise hysteresis in the flow-concentration relationship are widely noted for other land uses at different spatial scales (e.g. Ghadiri et al., 2001; Wilson et al., 2012; Gellis, 2013; Tena et al., 2014; Fang et al., 2015). This initial rise in sediment concentration can have adverse impact on ecosystems that are sensitive to certain threshold in sediment concentration. First flush, specifically the initial rapid rise followed by a gradual decline in concentration has been modelled by a combination of linear and exponential decay functions of the cumulative runoff volume in the context of urban stormwater runoff (Kim et al., 2005 and Stovin and Guymer, 2013). Parameter values of these models, while calibrated using field data, are difficult to interpret on physical grounds. Another framework known as GUEST (Griffith University Erosion System Template, Misra and Rose, 1996; Yu et al., 1997; Yu and Rose, 1999; Rose et al., 2011) can be used to characterise and explain this observed high sediment concentration during the early stage of a runoff event. Underlying GUEST is a system of partial differential equations (PDEs) that are meant to describe the dynamic exchange of sediments in suspension and those in a deposited layer for individual size classes (Hairsine and Rose, 1992). Fine sediments with small settling velocities are delivered to the catchment outlet or location of observation sooner than coarse sediments. This initial rise in sediment concentration occurs because the rate of exchange between suspended sediment and the deposited layer is slower for finer particles with lower settling velocities. Steady state is reached in the long run when the rate of sediment delivery and the size distribution of eroded sediments equilibrate with the rate of erosion and size distribution of the original soil matrix, respectively. In general, there are no exact solutions to the set of PDEs except for some extremely simple cases (e.g. Barry et al., 2010), and approximate solutions have been sought to capture the essential characteristics of the dynamic change in sediment concentration over time during a runoff event.

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List of Notations

The following symbols are used in the paper:

B_i	a dimensionless composite variable defined in Eq. (19)
c	sediment concentration for uniform grain-sized soil, ML^{-3}
c_i	sediment concentration for size class i, ML^{-3}
c_s	the steady state total sediment concentration, ML^{-3}
c_t	total sediment concentration, ML^{-3}
d_i	deposition rate for size class i per unit bed area, $ML^{-2}\,T^{-1}$
D	water depth, L
E_i	a composite term defined in Eq. (8), TL^{-1}
f_i	the mass fraction of size class i in the original soil matrix
f_{oi}	the mass fraction of size class i in the deposited layer
F	the fraction of excess stream power effective in entrainment and re-entrainment of soil, MT^{-3}
g	the acceleration due to gravity, $L\,T^{-2}$
H	the fractional shielding of the original soil matrix provided by the deposited layer
H_e	the Heaviside function defined by Eq. (27)
H_s	the steady state fractional shielding of the original soil matrix provided by the deposited layer
H_∞	an approximate expression for the steady state fractional shielding of the original soil matrix
i	the sequence number of sediment size class order by settling velocity from small to large
N	the total number of sediment size classes
I_1	the modified Bessel function of the first kind of order 1
J	the energy required per unit mass of sediment for entrainment, $L^2\,T^{-2}$
L	slope length, L
m	the mass of the deposited layer per unit bed area for uniform grain-sized soil, ML^{-2}
m_i	the mass of sediment for size class i of the deposited layer per unit bed area, ML^{-2}
m_t	the total mass of sediment in the deposited layer per unit bed area, ML^{-2} M the total mass of sediment in the deposited layer per unit bed area required to completely shield the original soil matrix, ML^{-2}
N	the number of size classes
q	unit discharge, $L^2\,T^{-1}$
r_i	entrainment rate by flow from the original soil matrix for size class i per unit bed area, $ML^{-2}\,T^{-1}$
r_m	the maximum total entrainment rate by flow from the original soil matrix per unit bed area when $H = 0$, $ML^{-2}\,T^{-1}$
r_{ri}	re-entrainment rate by flow from the deposited layer for size class i per unit bed area, $ML^{-2}\,T^{-1}$
r_r	the maximum total re-entrainment rate by flow from the original soil matrix per unit bed area when $H = 1$, $ML^{-2}\,T^{-1}$
S	slope gradient
t	time, T
$t*$	characteristic time scale for the existing approximate solution (Rose et al. (2007)), Eq. (11), T
$t**$	characteristic time scale for the new approximate solution, T
v_a	the average settling velocity of the original soil matrix, LT^{-1}
v_i	the settling velocity for size class i, LT^{-1}
w_1	function defined used in the convolution integral solution of Eq. (28)
w_2	function defined used in the convolution integral solution of Eq. (29)
x	distance downslope, L
α	a parameter defined by Eq. (30), T^{-1}
β	a dimensionless composite variable defined in Eq. (31)
δ	relative error of the approximate solution in %
ρ	the density of clear water, ML^{-3}
σ	the wet density of sediment, ML^{-3}
Ω	stream power, MT^{-3}
Ω_0	the threshold stream power for entrainment and re-entrainment of soil, MT^{-3}

Approximate solutions of the equations describing the dynamic erosion of soil under steady rainfall have been developed by Sander et al. (1996), Hogarth et al. (2011), and Parlange et al. (1999). In order to simplify solutions, these authors assumed that spatial gradients in sediment concentration downslope were negligibly small compared to temporal gradients. This assumption has been justified by Hogarth et al. (2004a,b) using numerical methods. Rose et al. (2007) employed similar assumptions to develop an approximate solution to describe sediment concentration as a function of time at the end of a slope segment under steady flow conditions without rainfall, and showed reasonably good agreement between the approximate solution and observed sediment concentration over time, as well as the settling velocity distribution of exported sediments. In addition, the approximate solution was compared with the numerical solution in terms of the temporal variation in sediment concentration, and it was found that the two agreed well in qualitative terms (Rose et al., 2007).

The accuracy of the approximate solution of Rose et al. (2007) has, however, never been quantitatively assessed. As an example to illustrate the discrepancy between the approximate solution of Rose et al. (2007) and the numerical solution using the same input data and parameter values, Fig. 1 shows the time series of the total sediment concentration at the end of a 5 m slope segment at 1 s time intervals. The steady state solution for the sediment concentration is 15.94 kg m^{-3} computed using Eq. (15) for this data set. For the approximate solution of Rose et al. (2007), the steady

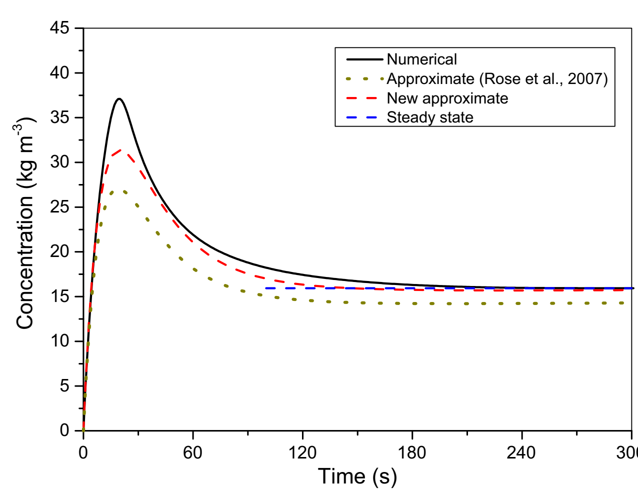

Fig. 1. A comparison of the numerical, approximate solution of Rose et al. (2007) and the new approximate solution using the same flow condition and soil characteristics.